

Optimal control of microalgae growth using linear quadratic regulator method with particle swarm optimization

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Abstract

Microalgae represent a significant and promising resource for a multitude of industrial applications. The lipids extracted from microalgae are used to produce biofuels, as dietary supplements for humans and animals, and as sources of vitamins or proteins in the pharmaceutical industry. Furthermore, microalgae serve as agents for carbon dioxide capture and wastewater treatment (Kumar et al., 2010). This study examines the optimal regulation of microalgal proliferation by controlling variables such as carbon dioxide and nutrients using the Linear Quadratic Regulator (LQR) methodology. Within the framework of the LQR approach, the weight matrices Q and R are commonly established through trial and error, a time-consuming process that does not guarantee a globally optimal solution. This manuscript advocates implementing Particle Swarm Optimization (PSO) to precisely determine the matrix values Q and R , thereby supplanting traditional manual tuning. PSO is a metaheuristic algorithm predicated on the collective behaviors observed in avian flocking and piscine schooling. Compared with the Firefly Algorithm previously applied to this problem, PSO offers a simpler velocity-position update mechanism, fewer algorithmic control parameters to calibrate, and a lower computational burden per iteration, while still maintaining a strong balance between exploration and exploitation of the search space. The principal novelty of this study lies in the application of PSO rather than the Firefly Algorithm to automatically tune the LQR weight matrices for the Thornton microalgae growth model, accompanied by an explicit, iteration-by-iteration account of the convergence behavior of the swarm, an aspect that has not been reported in prior LQR-based microalgae control studies. The use of PSO is expected to enhance microalgal growth, resulting in substantially increased biomass production. Simulation findings indicate that the LQR approach in isolation yields a 21.7% increase in algal concentration from the baseline concentration, whereas the LQR method, when augmented by PSO optimization, results in an approximate 64.5% enhancement in algal concentration from the initial level, thereby underscoring the advantage of PSO-assisted weight optimization over both the unoptimized LQR controller and the previously reported LQR–Firefly Algorithm combination.

Keywords: Microalgae, Linear Quadratic Regulator (LQR), Particle Swarm Optimization (PSO), Thornton growth model.

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INTRODUCTION

Indonesia is a vast nation endowed with extensive aquatic resources. Nevertheless, these plentiful natural assets have not been harnessed to their fullest capabilities. Concurrently, energy resources derived from fossil fuels continue to be excessively utilized, with their availability progressively diminishing. Consequently, there is an urgent need for alternative energy sources to supplant fossil fuels and meet the escalating demands of consumers. Microalgae constitute a promising renewable resource capable of

producing oil that can serve as biofuel, offering an environmentally sustainable energy solution (Zeng et al., 2011; Khoo et al., 2023). In addition to biofuel production, microalgae may also be converted into cosmetic products, dietary supplements, and a variety of other value-added commodities. Microalgae are relatively straightforward to cultivate and have the potential to yield microalgae oil at a rate of 16,103,453 kL/year utilizing merely 274 hectares of land (Abishek et al., 2014).

Harvesting microalgae during periods of low growth yields suboptimal biomass (Danquah et al., 2009). Therefore, optimizing microalgal growth is crucial for enhancing biomass production. Numerous studies have explored microalgae, ranging from optimizing control mechanisms in algal growth systems to lipid production in microalgae. Prior investigations elucidated models of lipid production in microalgae aimed at optimizing state variables (Prismahardi, 2018), in which the critical controlling factors were identified as substrate availability and carbon dioxide levels. In a similar vein, Mardlijah et al. (2015) examined the optimal control of nutrient flow to enhance algal growth (Hajar, 2015). Mairet et al. (2011) employed nitrogen limitation along with carbon dioxide as optimal control parameters for lipid production in microalgae.

Despite this body of work, a closer examination reveals a persistent research gap. Studies such as (Prismahardi, 2018), (Hajar, 2015), and (Mairet et al., 2011) establish optimal control formulations for microalgae growth but rely on manually selected weighting parameters within their respective control schemes, leaving the question of systematic, automated parameter tuning largely unaddressed. Setyowati and Mardlijah (Setyowati & Mardlijah, 2020) took an initial step toward closing this gap by coupling the LQR controller with the Firefly Algorithm to tune the weight matrices Q and R . Yet, the Firefly Algorithm itself involves several interdependent parameters (light absorption coefficient, attractiveness at zero distance, randomization parameter) that must be calibrated and that can slow convergence when poorly set (Prismahardi, 2018). To date, no published study on LQR-based microalgae growth control has examined Particle Swarm Optimization, with its comparatively simpler parameter set, as an alternative tuning mechanism, nor has any such study reported the iteration-level convergence trajectory of the optimizer used. This gap motivates the present study.

In the present study, the researchers investigate the optimal control of microalgal growth, using carbon dioxide and nutrient flows as control variables to maximize algal

concentration. This research utilizes the Linear Quadratic Regulator (LQR) methodology. LQR is a control strategy that incorporates trial-and-error approaches involving two weight matrices, Q and R . This method provides an objective function that evaluates system performance, thereby facilitating more straightforward analysis via numerical methods (Naidu, 2002).

One significant challenge associated with the conventional LQR approach is that its performance is heavily contingent upon the selected values of matrices Q and R , which are typically determined through trial and error, a labor-intensive process that may not produce globally optimal weight values. To mitigate this constraint, this paper proposes applying Particle Swarm Optimization (PSO) to autonomously and optimally determine the weight matrices Q and R . PSO is a population-based metaheuristic algorithm inspired by the social behaviors observed in bird flocking and fish schooling (Kennedy & Eberhart, 1995). Compared with the Firefly Algorithm employed in prior related research (Setyowati & Mardijah, 2020), PSO offers a competitive alternative optimization framework characterized by straightforward implementation and rapid convergence.

Several characteristics underlie the choice of PSO over the Firefly Algorithm for this tuning task. First, PSO updates each particle through a single velocity–position rule that combines inertia, a cognitive component (attraction to the particle’s own best position), and a social component (attraction to the swarm’s best position). In contrast, the Firefly Algorithm requires pairwise attractiveness computations between every pair of fireflies, increasing computational cost as the population grows. Second, PSO depends on a smaller set of control parameters (inertia weight ω , cognitive coefficient c_1 , and social coefficient c_2), which are comparatively easier to calibrate than the light absorption coefficient and randomization parameter required by the Firefly Algorithm. Third, the social-learning mechanism in PSO, whereby every particle is directly informed by the single global best solution, tends to accelerate convergence for low- to moderate-dimensional problems such as the two-parameter (Q, R) tuning task addressed here. These properties motivate the expectation that PSO can match or exceed the performance of the Firefly Algorithm while requiring less computational overhead and simpler parameter calibration. Beyond this bioprocess application, PSO-tuned LQR controllers have been reported to perform effectively across diverse engineering domains, including robust trajectory tracking of underactuated systems (Jerome & Vinodh Kumar, 2014) and

stabilization of self-balancing robots (Fang, 2014), lending further support to its suitability as a weight-tuning mechanism in the present study.

Based on the literature search summarized above, no published study on LQR-based microalgae growth control has yet examined Particle Swarm Optimization as an alternative to the Firefly Algorithm or to manual trial-and-error tuning; the novelty of this research therefore lies specifically in closing that gap by applying PSO to the automatic tuning of LQR weight matrices Q and R within the Thornton microalgae growth model, and in explicitly documenting the iteration-by-iteration convergence behavior of the swarm so that the optimization process, and not only its final result, is transparent and reproducible. Building on this novelty, the principal contributions of this study are threefold: (1) the formulation of an LQR-based optimal control scheme for microalgal growth in which the weight matrices are tuned automatically via PSO rather than manually; (2) a detailed, iteration-level account of PSO convergence including the number of iterations required and the iteration at which the global best solution stabilizes that is largely absent from prior LQR metaheuristic studies on microalgae control; and (3) a quantitative performance comparison among the uncontrolled system, the LQR-only controller, the previously reported LQR-Firefly Algorithm controller (Setyowati & Mardlijah, 2020), and the proposed LQR-PSO controller, providing evidence of the relative effectiveness of PSO as a weight-tuning mechanism for this class of bioprocess control problems.

METHODS

Mathematical Model of Algal Growth

This study utilizes a quantitative, simulation-driven approach to solve the optimal control problem of algal growth by integrating LQR with PSO. In this manuscript, the Thornton model is employed as the mathematical framework for algal proliferation (Thornton et al., 2010). Anthony Thornton formulated this model to elucidate algal growth in response to carbon dioxide concentration, nutrient availability, and glucose levels. According to the Thornton model, carbon dioxide diffuses into aquatic environments and is subsequently converted into glucose via photosynthesis. Nutrients present in industrial wastewater, in conjunction with glucose, facilitate the formation of algal biomass. It is posited that both the algal population and the glucose reserves

diminish due to mortality and harvesting practices. It is noteworthy that energy is not solely stored as glucose; rather, glucose is also metabolized into more complex compounds and lipids.

The production of algae is quantitatively modeled by considering the concentrations of algae (A), nutrients (M), glucose (S), and carbon dioxide (C) within pond ecosystems. Thornton postulates that the aquatic environment is well-mixed and that algal growth occurs at a relatively slow rate. The aforementioned concentrations are considered independent of other state variables and depend solely on temporal progression (t). The influx of nutrients and carbon dioxide is represented by the parameters I_m and I_c , respectively. Algae experience mortality at a rate denoted by death (d_r) and are harvested at a rate (h_r), both of which reduce algal numbers and glucose reserves. The rate of glucose production (α_s) is considered to be constant. The mathematical formulations characterizing microalgal growth are delineated as follows (Thornton et al., 2010):

$$\dot{A} = [\rho_{max}M(t)/(M(t)+M_{tur}^n)] \alpha_a S(t) - (d_r+h_r)A(t) \quad (1)$$

$$\dot{M} = -[\rho_{max}M(t)/(M(t)+M_{tur}^n)] k_2 \alpha_a S(t) + I_m(t) \quad (2)$$

$$\dot{S} = \alpha_s C(t) - [\rho_{max}M(t)/(M(t)+M_{tur}^n)] k_3 \alpha_a S(t) - (d_r+h_r)S(t) \quad (3)$$

$$\dot{C} = -k_1 \alpha_s C(t) + I^c(t) \quad (4)$$

Table 1. Variables and Parameters (Thornton et al., 2010)

Parameter	Description	Value
α_a	Rate constant for biomass growth	10.20
ρ_{max}	Maximal nutrient concentration inside algae	0.40
d_r	Algal death rate	0.46
h_r	Algal harvest rate	2.00
M_{tur}^n	Half saturation constant	4.00
I_m	Inflow of nutrient	4.20
α_s	Rate constant for photosynthesis	67.60
k_1	Conversion rate of CO ₂ into (CH ₂ O) ₆	0.40
k_2	Conversion rate of nutrients into dry algae	0.05
k_3	Conversion rate of (CH ₂ O) ₆ into dry algae	0.05
I^c	Inflow of carbon dioxide	4.00

Optimal Control of LQR

The Linear Quadratic Regulator (LQR) is a sophisticated control system that combines a dynamic system with a feedback gain mechanism, with the primary aim of optimizing algal proliferation. In this formulation, J denotes the cost functional to be optimized, expressed as a function of the state trajectory $x(t)$ and the control trajectory $u(t)$ over the control horizon: x is the state vector comprising the four state variables of the growth model, namely biomass A , nutrient M , substrate S , and carbon dioxide C , while u is the control (input) vector comprising the two manipulated variables, the nutrient inflow $I_m(t)$ and the carbon dioxide inflow $I^c(t)$. The objective function is articulated in the following manner:

$$J = (1/2) x^T P^c x + (1/2) \int_{[t_0, t^i]} (x^T Q^c x + u^T R^c u) dt$$

Here, t_0 denotes the initial time and t^i the terminal (final) time of the control horizon, so that the integral accumulates the running cost over the entire simulation period. The first term, $(1/2)x^T P^c x$, is the terminal cost evaluated at $t = t^i$, penalizing deviations of the final state from its desired value, so that P^c serves as the terminal (final-state) weighting matrix; the integral term is the running cost accumulated over the horizon, in which Q^c is the state weighting matrix that penalizes deviations of the state x from its desired trajectory, while R^c is the control weighting matrix that penalizes the magnitude of the control effort u .

wherein Q^c and P^c are characterized as semi-definite positive matrices ($Q^c \geq 0$, $P^c \geq 0$). Furthermore, R^c is identified as a positive definite matrix ($R^c > 0$). The principal aim of this investigation is to determine the optimal fluxes of nutrients and carbon dioxide that enhance algal density. For the microalgae control problem addressed here, no explicit terminal-state target is imposed, so the terminal-cost term is omitted ($P^c = 0$), and the generic weighting matrices Q^c and R^c are specialized to the scalar weight Q applied to the state vector x and the scalar weight R applied individually to each control input, I_m and I^c . This objective is mathematically delineated as the maximization of:

$$J = (1/2) \int_{[t_0, t^i]} (x^T Q x + R I_m^2(t) + R I^{c2}(t)) dt$$

where I_m and I^c denote the control variables, and R represents a component of the weighting control matrix.

Linear Quadratic Regulator Principle

The first step is to form the Hamiltonian function from the objective function:

$$H = x^T Q x + R I_m^2(t) + R I^{c2}(t) + \sum_{i=1}^4 \lambda_i (A x + B u).$$

In this expression, H is the Hamiltonian and $\lambda = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)^T$ is the costate (or adjoint) vector, with one costate variable associated with each of the four state equations defined in Section 1; the summation term $\Sigma \lambda_i (Ax + Bu)$ incorporates the system dynamics into the Hamiltonian via the state equation. According to Pontryagin's Minimum Principle (Kirk, 1998), the necessary conditions for optimality comprise the state equation, the stationarity condition $\partial H / \partial u = 0$, and the costate (adjoint) equation $d\lambda / dt = -\partial H / \partial x$; the latter describes how each costate variable λ_i , interpretable as the marginal (shadow) cost of its corresponding state variable along the optimal trajectory, evolves as a function of the sensitivity of the Hamiltonian to that state variable. Evaluating $d\lambda / dt = -\partial H / \partial x$ component-wise for the present model yields the following costate equations:

$$\begin{aligned} d\lambda_1 / dt &= (d_r + h_r) \lambda_1. \\ d\lambda_2 / dt &= [\alpha_a \rho_{max} S / (M + M_{tur}^n)] (-\lambda_1 + k_2 \lambda_2 + k_3 \lambda_3) + \\ &[\alpha_a \rho_{max} M S / (M + M_{tur}^n)^2] (\lambda_1 - k_2 \lambda_2 + k_3 \lambda_3). \\ d\lambda_3 / dt &= -[\alpha_a (\rho_{max} M / (M + M_{tur}^n)) (\lambda_1 - k_2 \lambda_2 - k_3 \lambda_3) - (d_r + h_r) \lambda_3]. \\ d\lambda_4 / dt &= -[\alpha_s \lambda_3 - k_1 \alpha_s \lambda_4]. \end{aligned}$$

Applying the stationarity conditions $\partial H / \partial I_m = 0$ and $\partial H / \partial I_c = 0$, the optimal control equations are:

$$\begin{aligned} I_m(t) &= -R^{-1} \lambda_2(t), \quad \text{with } 0 < I_m(t) \leq 4.2 \\ I_c(t) &= -R^{-1} \lambda_4(t), \quad \text{with } 0 < I_c(t) \leq 4. \end{aligned}$$

Because the growth dynamics are represented in linear state-space form and the cost functional is quadratic, the costate and state vectors are related through $\lambda(t) = P(t)x(t)$, where $P(t)$ is a symmetric, time-varying matrix; substituting this relation into the costate and state equations yields the (differential) Riccati equation, a matrix differential equation in $P(t)$ whose solution, together with the boundary condition at $t = t^i$, determines the optimal feedback gain relating the control input to the state. In this study, the costate vector $\lambda(t)$ is obtained by numerically solving the Riccati equation in MATLAB (Naidu, 2002).

Particle Swarm Optimization (PSO)

Particle Swarm Optimization (PSO) is a population-centric metaheuristic optimization technique first proposed by Kennedy and Eberhart in 1995 (Kennedy & Eberhart, 1995). This algorithm draws inspiration from the collaborative social dynamics observed in avian flocking and piscine schooling, wherein individual entities (particles)

modify their positions within the search domain based on their own experiences and those of proximate particles. In the context of this research, PSO is employed to enhance the optimization of the weight matrix parameters Q and R within the Linear Quadratic Regulator (LQR) methodology. It should be noted that the notation used in this section is distinct from that of the LQR formulation in Sections 2 and 3: here, $x_i(t)$ denotes the position of the i -th particle in the PSO search space at iteration t , where $i = 1, 2, \dots, n$ indexes the individual particles within the swarm and n is the swarm (population) size; each particle's position x_i is a candidate pair of LQR weighting values (Q, R) and is therefore unrelated to the state vector x of the microalgae growth model defined in Section 1. Correspondingly, $pBest_i$ denotes the best candidate (Q, R) pair found so far by particle i , and $gBest$ denotes the best candidate pair found so far by any particle in the swarm.

Velocity Update

At every iteration, each individual particle modifies its velocity in accordance with the equation:

$$v_i(t+1) = \omega \cdot v_i(t) + c_1 r_1 (pBest_i - x_i(t)) + c_2 r_2 (gBest - x_i(t)) \quad (5)$$

where ω signifies the inertia weight (within the range of $[0.4, 0.9]$), c_1 and c_2 represent the cognitive and social acceleration coefficients (established at a value of 2.0), r_1 and r_2 denote random variables distributed in the interval $[0, 1]$, $pBest_i$ refers to the personal best position attained by particle i , and $gBest$ indicates the global best position identified by the swarm.

Position Update

After velocity update, the new position of each particle is computed as:

$$x_i(t+1) = x_i(t) + v_i(t+1) \quad (6)$$

PSO Algorithm Parameters

In the present investigation, the swarm population is established at 30 particles, the upper limit for iterations is designated as 100, and the inertia weight diminishes in a linear fashion from 0.9 to 0.4 throughout the iterations to achieve a harmonious equilibrium between global exploration and local exploitation.

```
PSO Algorithm
Objective function  $f(x)$ ,  $x = (x_1, \dots, x_d)^T$ 
Generate initial population of particles  $x_i$  ( $i=1,2,\dots,n$ )
Initialize velocities  $v_i$  randomly
Evaluate  $f(x_i)$ ; set  $pBest_i = x_i$ ;  $gBest = \operatorname{argmax} f(pBest_i)$ 
while ( $t < \text{MaxIteration}$ )
  for  $i = 1 : n$  all particles
    Update velocity  $v_i$  using Equation (5)
    Update position  $x_i$  using Equation (6)
    Evaluate  $f(x_i)$ 
    if  $f(x_i) > f(pBest_i)$  then  $pBest_i = x_i$  end if
    if  $f(x_i) > f(gBest)$  then  $gBest = x_i$  end if
  end for
  Update inertia weight  $\omega$  (linear decay)
end while
Post-process results and visualization
```

Figure 1. Pseudo Code for PSO Algorithm (Kennedy & Eberhart, 1995)

RESULTS AND DISCUSSION

Simulation Results

This section elucidates the outcomes of the simulation for the microalgae growth model, in which nutrient availability and carbon dioxide levels are designated as pivotal control variables. The initial parameters for the simulation of algal proliferation are as follows: $A_0 = 3$ grams, $M_0 = 0.4$ grams, $S_0 = 10$ grams, and $C_0 = 5$ grams, with the simulation conducted over 100 days.

The Particle Swarm Optimization (PSO) algorithm was executed using a cohort of 30 particles across 100 iterations. Each particle signifies a prospective amalgamation of weight matrices Q and R . The swarm converges towards the globally optimal configuration that maximizes the objective function related to algal concentration.

Tracking the global best value of the objective function $f(gBest)$ at every iteration provides a direct measure of how quickly the swarm locates a satisfactory solution. At iteration 1, the global best objective value $f(gBest)$ stood at approximately 6.16098 grams. Over the following iterations the swarm explored the search space and progressively improved this value, and by iteration 6 the global best had risen to approximately 6.16776

grams, after which no further improvement was observed through iteration 40 (and, by extension, the remainder of the 100 allotted iterations). The swarm is therefore considered to have converged at iteration 6, with the global best value remaining unchanged for the subsequent iterations, indicating that the particles had collectively settled on the optimal combination of weight matrices Q and R well before the maximum iteration budget was exhausted. This rapid convergence is consistent with the relatively low dimensionality of the tuning problem (two scalar weights) and illustrates one of the practical advantages of PSO discussed in Section 1: a small number of iterations is sufficient to reach a stable optimum, reducing the computational cost of the weight-tuning procedure relative to exhaustive manual trial and error.

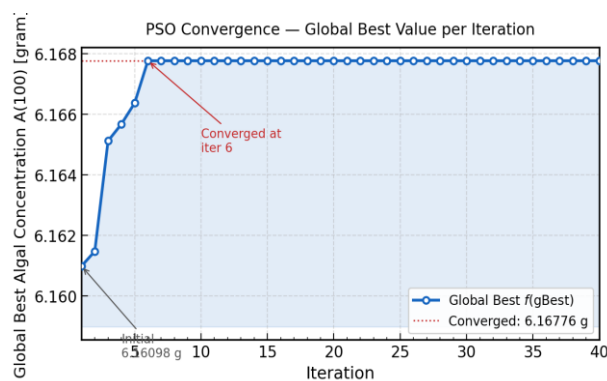


Figure 2. PSO Convergence Global Best Algal Concentration A(100) per Iteration

Figure 2 illustrates the configuration of Particle Swarm Optimization (PSO) particles as they converge toward a singular optimal point. The PSO simulation is employed to determine the optimal combination of the weight matrix parameters Q and R that maximizes the objective function, thereby facilitating the optimization of microalgae growth. The vertical axis, $A(100)$, represents the algal concentration attained at the end of the 100-day simulation horizon under the weight matrices encoded by the swarm's global best particle; because the PSO search is driven directly by this quantity, every upward step of the curve in Figure 2 corresponds to the discovery of a (Q, R) combination that yields a biologically more favorable steady-state algal biomass. The trajectory of the global best value rises sharply over the first few iterations, from approximately 6.16098 grams at iteration 1 to approximately 6.16776 grams at iteration 6, after which the curve flattens and remains unchanged through iteration 40, confirming convergence at iteration 6. In biological terms, this means that the swarm needed only a

handful of trial weight combinations to identify a control policy that channels nutrient and carbon dioxide inflows in a manner that sustains near-maximal algal biomass, after which additional search iterations yielded no further gains in final algal concentration.

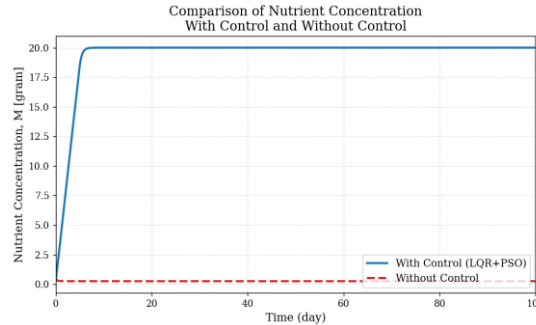


Figure 3. Comparison of nutrient concentration with and without control

Figure 3 shows that the nutrient concentration with LQR+PSO control rises steadily from its initial value of 0.4 grams, reaching approximately 7.67 grams by the second day and 11.34 grams by the third day, before gradually approaching a stabilized concentration of approximately 20 grams from around the fifteenth day onward through the 100th day of observation. In contrast, the nutrient concentration without control remains essentially flat near its initial value throughout the simulation horizon. This pattern reflects the underlying balance, expressed in Equation (2), between the nutrient inflow $I_m(t)$ prescribed by the PSO-tuned LQR controller and the rate at which nutrients are consumed in the conversion of substrate into algal biomass. The sustained rise indicates that the controller maintains a nutrient inflow that consistently exceeds the consumption demand of biomass synthesis, building up a nutrient reserve that, per Equation (1), elevates the saturation term $M(t)/(M(t)+M_{turn})$ toward its maximum and thereby sustains the elevated growth rate underlying the algal concentration trajectory shown in Figure 4.

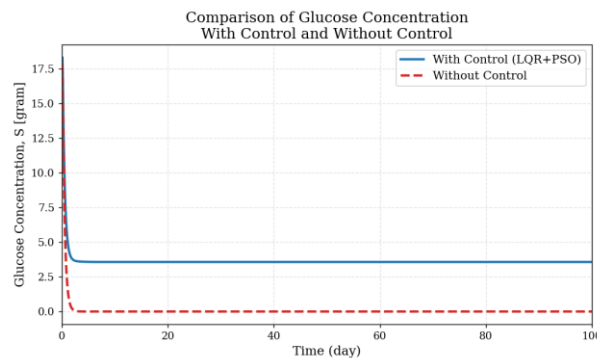


Figure 4. Comparative analysis of glucose concentration with and without the application of control measures.

Figure 4 illustrates that glucose concentration declines sharply during the first two days of the simulation under both the controlled and uncontrolled scenarios, starting from the initial value of 10 grams. Under the LQR+PSO controlled condition, glucose concentration falls to approximately 5.34 grams by the first day and 3.76 grams by the second day, after which it gradually settles to a stabilized concentration of approximately 3.57 grams from around the third day through the one-hundredth day of observation. Under the uncontrolled condition, glucose is depleted even more rapidly, approaching zero within the same initial period. This observed decline in glucose concentration is attributable to the utilization of glucose as a source of energy during the processes of algal formation and photosynthesis. Referring to Equation (3), glucose is generated through the photosynthetic conversion of carbon dioxide (the term $\alpha_s C(t)$) while simultaneously being consumed in proportion to nutrient availability as algal biomass is synthesized; the steep initial drop followed by stabilization at a markedly higher plateau than the uncontrolled case indicates that, under the PSO-tuned LQR controller, the steadily rising nutrient pool shown in Figure 3 sustains a higher glucose consumption rate that nonetheless settles into a quasi-equilibrium with the carbon-dioxide-driven photosynthetic supply, in turn maintaining the elevated steady growth of algal concentration observed in Figure 5.

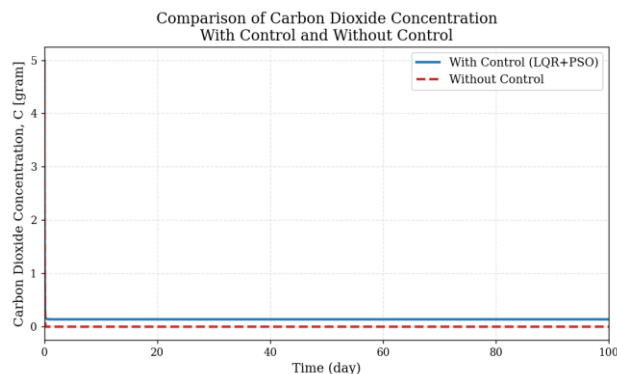


Figure 5. Comparison of carbon dioxide concentration with and without control

Figure 5 illustrates that under the LQR+PSO control framework, carbon dioxide concentration falls sharply from its initial value of 5 grams and stabilizes at approximately 0.139 grams from the early days of the simulation through day 100, with no subsequent rebound. This rapid and sustained depletion is attributable to the consumption of carbon dioxide during the synthesis of glucose via photosynthesis. As described by Equation (4), the carbon dioxide pool is depleted at a rate proportional to the photosynthesis constant $k_1 \alpha_s$ and replenished only by the controller-prescribed inflow $I^c(t)$; because the intrinsic

depletion rate $k_1\alpha_s$ is large relative to the admissible inflow bound on $I^c(t)$, the controller cannot prevent carbon dioxide from settling at a low steady-state level. The pronounced and sustained reduction in carbon dioxide concentration shown in Figure 5 therefore, indicates that the PSO-tuned LQR controller favors directing carbon dioxide rapidly into the photosynthetic pathway, supplying the glucose pool depicted in Figure 4 and, in turn, supporting the sustained algal biomass accumulation shown in Figure 6, rather than allowing carbon dioxide to accumulate unused within the system.

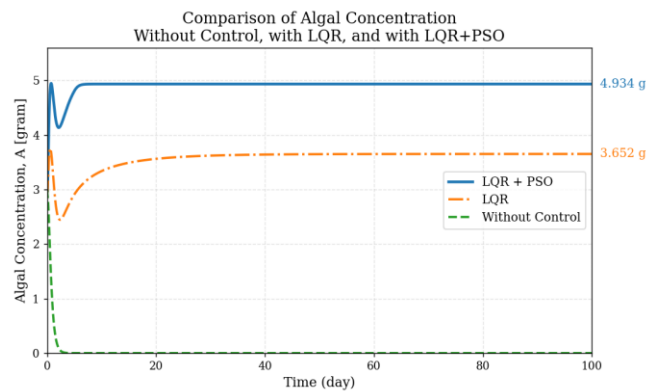


Figure 6. Comparison of algal concentration without control, with LQR control, and LQR+PSO

Figure 6 presents a comparative analysis of microalgal concentrations across three scenarios: no control, Linear Quadratic Regulator (LQR) control, and LQR with Particle Swarm Optimization (PSO) control. In the absence of any control measures, the algal concentration declines continuously. Conversely, the application of LQR control alone increases algal concentration to 3.755 grams, which subsequently stabilizes at 3.652 grams by the end of day 100, representing a 21.7% increase relative to the initial concentration. In the scenario involving LQR coupled with PSO optimization, the algal concentration increases to 4.947 grams and stabilizes at approximately 4.934 grams, representing an increase of approximately 64.5% over the initial concentration of 3 grams. This outcome demonstrates that the PSO-optimized LQR strategy outperforms the standalone LQR controller and offers a modest improvement over the Firefly Algorithm-LQR combination (63.3%) reported in prior research (Setyowati & Mardlijah, 2020). It is also worth noting that the LQR and LQR+PSO trajectories each exhibit a brief, sub-day dip immediately after $t=0$ before climbing to their respective peaks; this transient arises because the carbon dioxide pool decays intrinsically much faster (Equation 4, rate

constant $k_1\alpha_s$) than the admissible controller inflow can replenish it, so glucose production momentarily lags before the rising nutrient inflow shown in Figure 3 takes hold and drives algal growth upward.

Biologically, the trajectories in Figure 6 are consistent with the resource-flow s established by Figures 3–5: the PSO-tuned LQR controller regulates nutrient and carbon dioxide inflows (Equations 2 and 4) to keep the glucose pool (Equation 3) sufficiently supplied to sustain biomass synthesis through Equation (1), thereby allowing algal concentration to keep rising rather than collapsing as in the uncontrolled case. The margin between the LQR-only curve and the LQR+PSO curve in Figure 6 is, in effect, a direct biological consequence of the swarm having located, by iteration 6 of the PSO run shown in Figure 2, a combination of weight matrices Q and R that channels these resource flows more efficiently into algal biomass than the manually tuned LQR weights. The 1.2-percentage-point margin over the previously reported LQR+Firefly result (Setyowati & Mardlijah, 2020) further suggests that, for this two-parameter tuning problem, PSO is at least as competitive as the Firefly Algorithm at directing the controller toward weight combinations that favor sustained biomass accumulation.

Table 2. Comparison of Simulation Results

Method	Peak Algal Concentration (gram)	Increase from Initial (%)
Without Control	< 3 (decreasing)	-
LQR Control	3.755	21.7%
LQR + PSO	4.947	64.5%
LQR + Firefly	4.921	63.3%

CONCLUSION

The concentration of microalgae, in the absence of control measures, declines continuously over time due to natural mortality and losses incurred during harvesting. The implementation of the Linear Quadratic Regulator (LQR) control methodology results in a notable 21.7% increase in microalgae concentration relative to the initial concentration. Moreover, the synergistic application of the LQR method in conjunction with Particle Swarm Optimization (PSO) for the optimization of weight matrices yields an increase in microalgae concentration of approximately 64.5% from the initial concentration, thereby surpassing the outcomes achieved through the LQR-only strategy

and showing a modest improvement over the previously documented LQR+Firefly Algorithm integration, which reported an increase of 63.3%. These findings substantiate the assertion that PSO is an efficacious and competitive alternative for calibrating LQR weight matrices in optimizing microalgae growth.

Prospective investigations may consider integrating temperature and light intensity into the microalgae growth model further to enhance biomass production under more realistic cultivation scenarios. In addition, a comparative analysis involving alternative metaheuristic algorithms, such as Genetic Algorithms, Grey Wolf Optimizer, or Whale Optimization Algorithm, could yield comprehensive insights into the most effective optimization strategies for addressing bioreactor control challenges.

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