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Argumentation ability of eighth-grade students on Pythagorean theorem material using Toulmin's argumentation model

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Abstract

It is known that students' argumentation skills remain in the low category, and it is important for teachers to improve the teaching and learning process to foster their development. Before making improvements, it is necessary to assess each student's argumentation ability to determine appropriate steps to enhance the teaching and learning process, ensuring that every student's argumentation skills improve. Therefore, this study aims to describe students' argumentation skills on the Pythagorean Theorem using Toulmin's Argumentation Scheme (claim, data/grounds, warrant, backing, qualifier, rebuttal) among eighth-grade students in junior high school. The research employed a descriptive approach. The research subjects were four eighth-grade students, purposively selected from one class. Data were collected through: (1) contextual problem-solving tasks on the Pythagorean Theorem designed to elicit argument components, (2) argumentation worksheets based on Toulmin's scheme, (3) clinical interviews to explore students' reasoning, and (4) observation and document analysis. Data analysis was conducted through data reduction, display, and conclusion drawing, using coding techniques to identify the occurrence and quality of each Toulmin component, then grouping them into low, medium, and high argument quality categories. The results showed that one student had high argumentation ability, two had moderate, and one had low.

Keywords: Pythagorean Theorem; Toulmin's scheme; argumentation skills: eighth-grade students; descriptive

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INTRODUCTION

Mathematics is a crucial subject that plays a vital role in developing students'logical, systematic, and analytical thinking skills (Carolin et al., 2025) Mathematical argumentation skills are a fundamental component of students' logical and critical thinking in mathematics learning. Argumentation refers to a statement supported by objective data and facts to justify or influence a conclusion (Sadieda, 2019). In the context of mathematics, mathematical argumentation is the ability to provide reasons, data, and justification to support or reject a claim related to a mathematical problem, both orally and in writing (Putra et al., 2022). This ability enables students to construct a proper understanding of mathematical concepts and to communicate their reasoning effectively.

Furthermore, argumentation is closely related to critical thinking and problem-solving skills, which are essential competencies in 21st-century learning (Duhita et al., 2021).

The importance of mathematical argumentation has been widely emphasized in previous studies. Students who possess strong argumentation skills can present logical reasoning, supported by relevant data and theoretical justification (Soekisno, 2015). However, argumentation skills do not develop automatically and require deliberate instructional efforts (Duhita et al., 2021). Therefore, the development of students' mathematical argumentation skills should become a key focus in mathematics learning.

Despite its importance, empirical evidence indicates that students' mathematical argumentation skills remain relatively low. Research indicates that students often struggle to construct complete arguments, particularly in providing justification and rebuttals (Resmi et al., 2021). Previous studies also reveal that students at the secondary school level generally reach only basic levels of argumentation, limiting themselves to stating claims and presenting simple information without deeper reasoning (AR et al., 2018). Similarly, Lubis and Lubis (2024) found that even students with high initial ability tend to have weak rebuttal components, indicating difficulty rejecting incorrect statements. Other studies report that students can express opinions but fail to provide sufficient reasoning and evidence to support them (Agustiningsih et al., 2021) and that overall argumentation performance remains below expected standards (Syerliana et al., 2018). These findings suggest that students have not yet developed comprehensive skills in mathematical argumentation.

Several studies have examined factors influencing students' mathematical argumentation skills, such as gender differences (Sholihah et al., 2021), levels of self-actualization (Khoirunnisa, 2017), and prior mathematical ability (Dian, 2015). These studies indicate that students' argumentation skills vary across individuals. However, most of these studies focus on identifying influencing factors rather than analyzing in depth the structure and quality of students' argumentation using a specific analytical framework.

To analyze students' argumentation more systematically, the Toulmin argumentation model can be utilized. This model consists of six components: claim, data, warrant, backing, qualifier, and rebuttal, which provide a comprehensive framework for examining the structure and quality of arguments (Sadieda, 2019). The use of this model enables researchers to analyze not only students' final answers but also the reasoning processes underlying them (Sholihah et al., 2021). Therefore, the Toulmin model is considered appropriate for analyzing both formal and informal argumentation in mathematics learning (Lubis & Lubis, 2024).

Geometry is often perceived as a challenging subject due to students'limited understanding of its abstract concept (Tralisno & Alfi, 2025) .One mathematical topic that requires strong argumentation skills is geometry, particularly the Pythagorean Theorem. This topic involves reasoning, justification, and logical thinking processes, which require students to construct valid arguments in problem-solving (Putri Anasis & Alyani, 2021). However, previous studies indicate that students still experience difficulties in understanding and applying the Pythagorean Theorem (Nurkhaeriyyah et al., 2018). For instance, research by Novia et al. (2017) shows that students' average score on Pythagorean material remained below the minimum completeness criteria, indicating a need for improvement in learning outcomes. This difficulty may be related to students' limited argumentation skills in explaining and justifying their solutions (Nurafni et al., 2018).

Based on the above discussion, although numerous studies have investigated students' mathematical argumentation skills, there remains a lack of in-depth analysis of how students construct arguments using the Toulmin Argumentation Model, particularly in the context of the Pythagorean Theorem. This indicates a research gap that needs to be addressed.

Therefore, this study aims to describe students' mathematical argumentation skills on the Pythagorean Theorem topic using the Toulmin Argumentation Model. The findings of this study are expected to provide a more comprehensive understanding of students' argumentation structures and contribute to the development of more effective mathematics learning strategies.

METHODS

This study employed a qualitative descriptive design to describe students' mathematical argumentation skills regarding the Pythagorean Theorem, using the Toulmin Argumentation Model. This design was selected because the study focuses on exploring and interpreting the structure of students' arguments in depth, while scoring and categorization were used as supporting tools to describe students' argumentation ability.

The participants of this study were four eighth-grade students from one junior high school selected through purposive sampling. The selection was based on specific criteria: students' ability to communicate mathematical reasoning both orally and in writing, as well as their representation of different levels of mathematical ability (high, medium, and low). This was intended to obtain diverse data reflecting variations in students' argumentation skills.

Data were collected using three instruments: a written test, interviews, and documentation. The written test consisted of problems on the Pythagorean theorem designed to elicit students' argumentation using Toulmin's components: claim, data, warrant, backing, qualifier, and rebuttal. Semi-structured interviews were conducted after the test to explore students' reasoning processes in greater depth and clarify their written responses. Documentation, including students' answer sheets, interview recordings, and field notes, was used as supporting data. Experts in mathematics education validated all instruments to ensure content validity and alignment with the Toulmin Argumentation Model, and revisions were made based on their suggestions.

A scoring rubric was developed based on the six components of the Toulmin Argumentation Model. Each component was assessed using a scale to indicate the completeness and accuracy of students' arguments. The data analysis process involved several stages, including data reduction, coding, scoring, categorization, and interpretation. Students' responses from both written tests and interviews were coded by identifying segments corresponding to Toulmin's components (claim, data, warrant, backing, qualifier, and rebuttal). Each component was then scored according to the rubric, and the total scores were used to classify students' argumentation ability into high, medium, and low categories based on predetermined intervals. The results were interpreted to describe the characteristics of students' argumentation patterns in solving Pythagorean Theorem problems.

Table 1. Rubric for Assessing Argumentation Using the Toulmin Model

Toulmin Component	Description of Expected Student Response	Score 0	Score 0.5	Score 1
Claim	Students state the opinion/position (one or more statements) explicitly	No claim / no opinion stated.	There is a claim, but it is not clear or not firm.	The claim is clear, specific, and consistent.

	in each statement of the problem.			
Data / Ground	Students show calculations or information using the Pythagorean Theorem to support their claim.	No data / wrong total.	There is calculation, but it is not complete or contains minor errors.	Calculations are complete, correct, and in accordance with the concept.
Warrant	Students explain why the calculation or information can be used to support the claim (relationship between data and claim).	No explanation.	There is an explanation, but it is still unclear or incomplete.	The explanation is clear, logical, and links the data with the claim.
Backing	Students provide reasons or additional theoretical support that strengthens the warrant, for example, properties of sides, general rules of the Pythagorean Theorem, or relevant properties.	No backing provided.	Backing is present, but it is still weak or less relevant.	Backing is relevant and strongly supports the warrant.
Qualifier	Students state the conditions or limits under which the claim can be made (for example, measurement accuracy, type of triangle, units, and so on).	No qualifier provided.	There is a qualifier, but it is less relevant.	The qualifier is clear, relevant, and logical.
Rebuttal	Students consider counterarguments or alternative cases (for example, if the measurement is wrong, if the unit is different, or other conditions that may cause the claim not to hold).	No rebuttal provided.	There is a rebuttal, but it is not clear or not well developed.	The rebuttal is logical, clear, and anticipates counterarguments appropriately.
Maximum Total Score: 12				

Score Interpretation:

- 0 – 4 = Low (Incomplete argument, only stating the claim whitout reasons)

- 5 – 8 = Medium (Some components exist, but the claim + data + warant are still incomplete)
- 9 – 12 = High (Complete argument, all Toulmint coponents are fullifed properly)a

To ensure data trustworthiness, triangulation was used by comparing findings from written tests, interviews, and documentation. In addition, coding consistency was maintained through careful review of the classification of Toulmin components. Overall, the data were analyzed using descriptive qualitative analysis supported by quantitative scoring to provide a comprehensive description of students' mathematical argumentation skills.

RESULTS AND DISCUSSION

In this stage, the researcher evaluates the students' argumentation skills based on the results of written tests and interviews. Based on the students' test results, the researcher analyzed the scores according to the rubric (Table 1) to calculate the average score and then categorized their problem-solving skills. The percentage of problem-solving test results for eighth-grade students (23 students) is shown in the Table 2.

Table 2. Test Result Percentage

Score range	Frequency	Percentage	Category
0 – 4	2	9%	Less
5 – 8	8	35%	Medium
9 – 12	13	56%	Height

From the test result data in Table 1, it can be observed that most subjects have been able to convey their arguments; the average score is 9, which is in the high category. With an average score of 9, the entire subject can work up to the fourth indicator, namely backing.

Analysis of Argumentative Ability in Answer to Question Number 1

Question number 1 includes six indicators of argumentation ability: claim, data, warrant, backing, qualifier, and rebuttal. Question 1 discusses the ladder leaning against the wall. The first indicator appears when the student expresses an opinion/position (agreeing or disagreeing with the statement on the question). The second indicator appears when the student uses the Pythagorean Theorem in a calculation to support his claim. The third indicator arises when the student explains why the calculation can be used to support a claim (the relationship between data and claims). The fourth indicator appears when the student gives additional reasons that reinforce the warrant, such as the nature of a right triangle, a general Pythagorean rule, or the quadrant nature of an oblique side. The fifth indicator appears when the student mentions conditions that could affect the results (e.g., measurement errors, unscaled drawings, sloped floors, etc.). The sixth indicator appears when the student responds to the opponent's argument (e.g., "true can be mismeasured, but if the right measure is used, the result still applies"). The question is as follows.

1. A worker puts a 13 m long ladder attached to the wall. The distance of the bottom of the stairs to the wall is 5 m.Tono said, "The height of the wall that the stairs reach must be 12 m, because this triangle is right."
   However, Rika hesitated: "It is not necessarily 12 m. It could be less, especially if the floor is uneven."

a. Do you agree with Tono or Rika?
b. Do **the math** to prove who's right.
c. Explain **why the calculation can be used** to determine height.
d. Give additional reasons why the Pythagorean rule is relevant in this case.
e. Write down what conditions can make the calculation results incorrect.
f. Respond to Rika's opinion about uneven floors.

Figure 1. Question Number One

The following is an analysis of the results of the research subject's answers in solving question number 1

a. MYS subject answer results

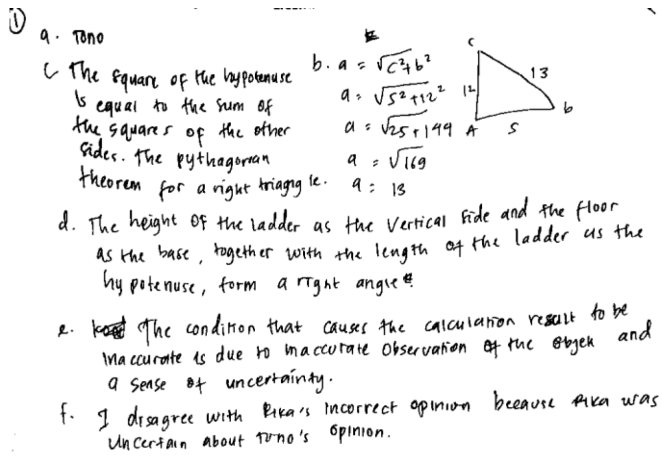

Figure 2. MYS's Answer to Question Number 1

MYS subjects can solve problem 1 accurately. MYS meets all six indicators of mathematical argumentation ability. The following is the answer to question 1 in the MYS subject. For the first indicator, the MYS subject has met it, so he gets a score of 1. This is evident in the MYS subject, who agreed with Tono. The MYS subject also fulfilled the second indicator by showing the calculation using the Pythagorean theorem formula and supposing the existing sides by naming a, b, and c. It then correctly performed the calculation, resulting in a perfect score of 1. The MYS subject also fulfilled the third indicator by explaining why the formula can be used and why it is true, and it received a score of 1. The MYS subjects also met the fourth indicator by providing additional reasons that could strengthen the previous answer. They stated that the wall was the upright side, the floor was the bottom side, and the stairs were the sloping side. So the MYS subject also received a score of 1 for the answer to question 1d. MYS also received a score of 1 on question 1e because it met the fifth indicator by mentioning conditions that could alter the results, namely, inaccurate observation of objects and feelings of hesitation. However, this reason is not very standard. The MYS subject also met the sixth indicator because it could respond to the arguments given; therefore, it received a score of 1 as well. This MYS subject has understood the material on the Pythagorean theorem and has met all indicators of argumentation ability; thus, the argumentation ability of MYS subjects is included in the high category. The results of the interview show the following:

P	:	Whose opinion do you agree with?
MYS	:	Tono
P	:	How do you prove your opinion correct?
MYS	:	by performing calculations using the Pythagorean formula
P	:	Why can the formula be used for this problem?
MYS	:	Because the square of the slanted side is equal to the sum of the other 2 sides
P	:	What does your data/calculations have to do with the claims you make?
MYS	:	height as the upright side, the floor as the bottom side, and the stairs as the sloping side
P	:	What conditions could make the calculation inappropriate?
MYS	:	inaccurate object observation
P	:	What if the floor is uneven
MYS	:	The calculation will be different.

b. Results of the ZS subject

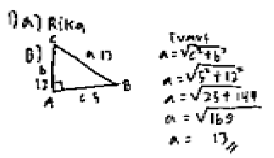
c. Because the ladder forms the hypotenuse and uses the Pythagorean formula.
d. Because the ladder is leaning against the wall and forms a right angle.
e. If the floor is slanted or the ladder does not lean perpendicular to the wall, then the result will be different.
f. Don't follow Rika because she is wrong; the correct one is Tono.

Figure 3. ZS's Answer to Question Number 1

The ZS subject answered all the number one questions and met all the indicators of argumentation ability. The ZS subject answered question 1a with the wrong answer, so he did not get a score, but he met the first indicator because he was able to express his opinion. In question 1b, the ZS subject correctly performed the calculation and met the second indicator, so he received a score of 1. The ZS subject also met the third indicator by giving the correct reason: the ladder formed an oblique side, and the Pythagorean formula was used to obtain the answer to the 1c question, so it got a score of 1. The ZS subject can also give a reason for the 1d problem: because the ladder is attached to the wall and forms a right angle. The ZS subject also gets a score of 1 for this problem. The ZS subject also met the fifth indicator and received a score of 1 for question 1e because the answer was correct: when the floor is tilted, or the stairs are not perpendicular, the results differ. The ZS subject also met the sixth indicator and answered the 1f question by responding to the opponent's argument, even though the reason was not very convincing. It can be concluded that the argumentative ability of ZS subjects falls into the high category. The following are the results of the interview with the subject ZS.

P : Whose opinion do you agree with?
ZS : Tono
P : Why did you answer Rika yesterday?
ZS : After I remembered again, I made a mistake yesterday, I wrote it wrong
P : Why can the formula be used for this problem?
ZS : because the stairs are like sloping sides
P : What does your data/calculation have to do with the claim you made?
ZS : because the ladder is attached to the wall and forms right angles so that the Pythagorean formula applies
P : What conditions can make the calculation inappropriate?
ZS : Floor/Sloping Wall
P : So whose opinion are you sure of?
ZS : Request

c. ABS subject answer results

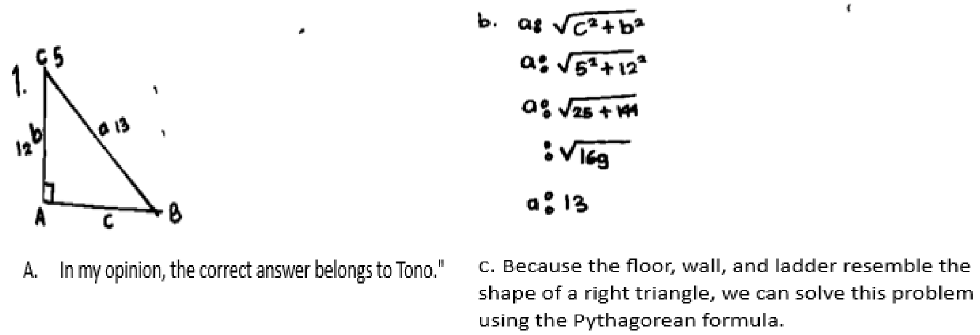

Figure 4. ABS's Answer to Question Number 1

ABS subjects were able to answer only 3 out of 6 questions for question 1. ABS subjects met only 3 indicators, including the first claim, which required an opinion agreeing with number one, namely tono. The second indicator of data is the correctness of calculations. The third indicator is the warrant for the subject's reason from the data provided, or the reason the subject uses the Pythagorean formula: namely, because the floor, wall, and staircase resemble a right triangle. So that the ABS subject only got a score of 3 for the overall number 1 question. Based on the ABS subject's answers, his argumentative ability falls into the medium category. The following are the results of the interview with ABS subjects:

P : Whose opinion do you agree with?
ABS : Agree with Tono
P : Why do you agree with Tono?
ABS : because after I calculate using the correct Pythagorean formula of tono
P : Why can the formula be used for this problem?
ABS : Because the floor, walls, and stairs resemble the shape of a right triangle so it can use the Pythagorean formula.
P : Why don't you do the 1D-1F problem?
ABS : Running out of time because I don't understand 1D questions
P : If it's about 1e and 1f, do you understand what it means?
ABS : Yes
P : What if the floor is uneven/sloping
ABS : The calculation will not be appropriate

d. BP subek answer results

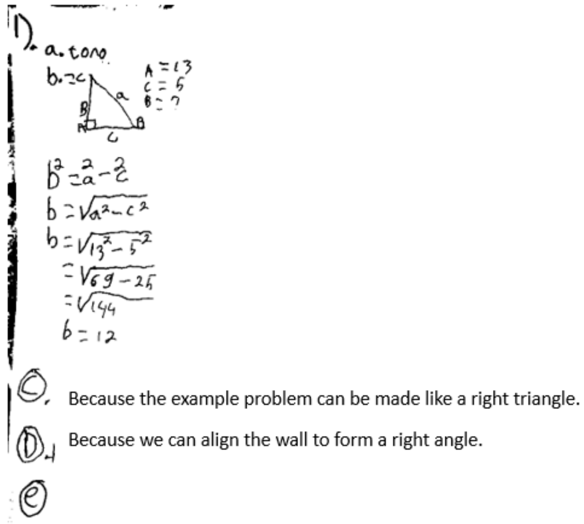

Figure 5. BP's Answer to Question Number 1

BP subjects answered only 4 of 6 questions on question 1. BP subjects met the first indicator by expressing agreement with the Tono and receiving a score of 1. BP subjects also met the second indicator by performing the calculations correctly and earning a score of 1 on question 1b. BP also answered questions 1c and 1d, which means it meets indicators 3 and 4, even though the argument is not accurate. Therefore, it can be drawn that the conclusion from the results of the BP subject's answers is included in the category of insufficient argumentation ability. The following are the results of the interview with BP subjects:

P : Whose opinion do you agree with?
BP : Request
P : Why do you use that formula to answer question 1b?
BP : Because I am studying Pythagoras, so I use the Pythagorean formula.
P : Why don't you answer the whole question?
BP : Confused as to how to explain it

Analysis of Problem-Solving Abilities in Question Number 2

Question number 2 includes six indicators of argumentation ability: claim, data, warrant, backing, qualifier, and rebuttal. Question 2 concerns a triangle with sides measuring 8 cm, 13 cm, and 17 cm. The first indicator appears when the student expresses an opinion/position (agreeing or disagreeing with the statement on the question). The second indicator appears when the student uses the Pythagorean Theorem in a calculation to support his claim. The third indicator arises when the student explains why the calculation can be used to support a claim (the relationship between data and claims). The fourth indicator appears when the student provides additional reasons that reinforce the warrant, such as the right triangle property, the Pythagorean theorem, or the slanted quadratic property. The fifth indicator appears when the student mentions conditions that can make the results different (e.g., making Rani's statement true/false). And the sixth indicator appears when the student responds to the opponent's argument (e.g. "true can be mismeasured, but if the right measure is used, the result still applies"). Question 2 is described in Figure 6.

2. Rani stated that triangles with side lengths of 8 cm, 13 cm, and 17 cm are right triangles. However, Dika disagreed. He argued that not all triangles with these sides must be right-angry, they could be blunt triangles, taper triangles, equilateral triangles, or arbitrary triangles.

a. Do you agree with Rani or Dika's opinion?
b. Show **calculations** that support your opinion!
c. Explain **why the calculation proves or refutes** Rani's statement.
d. Give an additional reason why the rule always applies to right triangles.
e. If there are special conditions that make Rani's statement true/false, mention it.
f. Responding to Dika's argument: is it really possible to mismeasure? What is a triangle called?

Figure 6. Question Number Two

The analysis of the results of the research subject's answers to question number two are described as follows.

a. MYS subject answer results

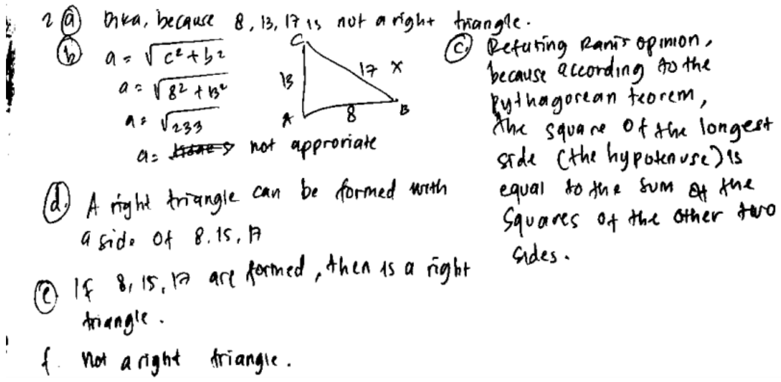

Figure 7. MYS's Answer to Question Number 2

The MYS subject solved problem number 2 well and accurately, and MYS met all six indicators of mathematical argumentation ability. The following is the answer to question 2 in the subject MYS. For the first indicator that the MYS subject has met, he receives a score of 1 because he expresses his opinion correctly, accompanied by reasons. The MYS subject also fulfilled the second indicator by showing the calculation using the

Pythagorean theorem formula and by giving an example of the existing sides by naming a, b, and c. The MYS subject also correctly calculated and gave a Decision that it was not suitable, so it got a perfect set, namely 1. The MYS subject also fulfilled the third indicator by explaining why the formula can be used and why it is true, and it received a score of 1. MYS subjects have also met the fourth indicator by giving the right answers. So the MYS subject also received a score of 1 for the answer to question 1d. MYS also got a score of 1 on question number 1e because it met the fifth indicator by mentioning the conditions that could make the result a right triangle. The MYS subject also met the sixth indicator because it could respond to the arguments given; therefore, it received a score of 1 as well. This MYS subject has understood the material of the Pythagorean theorem and has fulfilled all indicators of argumentative ability. The results of the interview are shown as follows.

P : Whose opinion do you agree with?
MYS : Dika, because Rani's calculation is wrong
P : How do you prove your opinion correct?
MYS : by performing calculations using the Pythagorean formula
P : What is the result of the calculation?
MYS : the square of the slanted side is not equal to the square of the other sides.
P : What should be the value of the side?
MYS : 8,15,17
P : if not 8,15,17 what is a right triangle?
MYS : no

b. Results of the ZS subject

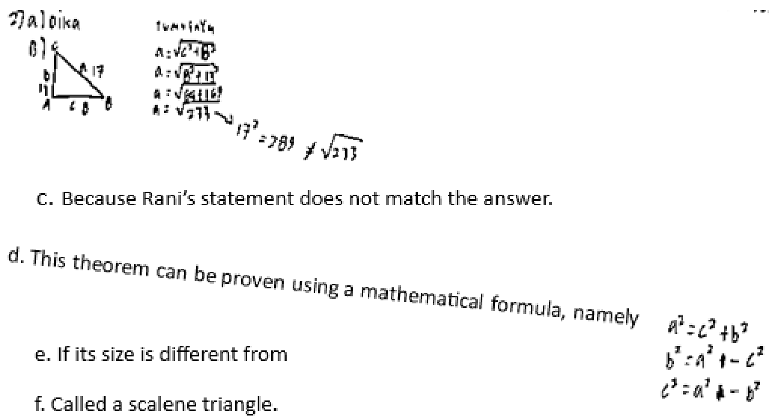

Figure 8. ZS's Answer to Question Number 2

The ZS subject answered all the number one questions and met all the indicators of argumentation ability. The ZS subject answered the 2a question correctly, namely dika, so it received a score of 1. And has met the first indicator because it can express opinions.

In question 2b, the ZS subject performed the calculation correctly and met the second indicator, so he got a score of 1. The ZS subject also met the third indicator by giving the correct reason: Rani's statement was not in accordance with the answer. The ZS subject can also give reasons for the number 1d problem and rewrite the formula. The ZS subject can also answer questions 1e and 1f, even though it is not very complete. The following are the results of the interview with the subject ZS.

P : Whose opinion do you agree with?
ZS : Düsseldorf
P : Why do you agree with Dika?
ZS : because Rani's calculation is wrong
P : Why is it wrong?
ZS : because the end result is not the same
P : Do you think it's a right triangle?
ZS : no, any triangle

c. ABS subject answer results

a. I agree more with Dika's opinion.

~~b. $17^2 = 13^2 + 8^2$~~

b. $17^2 = 8^2 + 13^2$

~~17×17~~

$289 = 64 + 169$

$289 = 233$

c. Refuting it because the calculation is wrong

d. Because the Pythagorean theorem only applies to right triangles.

e. The side should be 8,15,17

Figure 9. ABS's Answer to Question 2

ABS subjects answered only 5 of 6 questions for question 2. ABS subjects met only 5 indicators, including the first claim, which was met by giving an opinion agreeing with number 2a, namely, agreeing with Dika. The second indicator of data is obtained by making calculations, and the results show differences. The third indicator is the warrant for the reason the subject of the data provided, or the reason the subject agrees with Dika, because the calculation is wrong. The subject also answers the 1d question by stating the Pythagorean property, which applies only to right triangles. The ABS subject can also give answers that should be correct. The following are the results of the interview with ABS subjects.

P : Whose opinion do you agree with?
ABS : Agree with dika
P : Why do you agree with Dika?

ABS : because if the sides are 8,13,17 then it is not a right triangle
P : Why not a right triangle?
ABS : Because if calculated with the Pythagorean formula the result is not suitable
P : What should be the value of the sides?
ABS : 8,15,17
P : Why don't you answer 2f?
ABS : Out of time

d. BP subject answer results

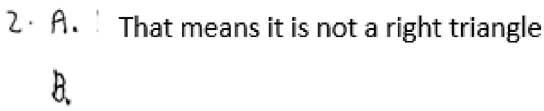

Figure 10. BP’s Answer to Question 2

The BP subject answered only 1 of 6 questions: question 2. The BP subject did not meet the first indicator because he did not express an opinion that aligned with anyone, so the BP subject did not score at all on question 2. The following are the results of the interview with the subject of BP.

P : Why do you only answer number 2A?
BP : because it's difficult
P : What makes it difficult for you to answer?
bp : I forgot and didn't pay much attention to previous learning

Based on the results of written tests, interviews, and documentation, it is possible to get an idea that the argumentation skills of grade VIII B students of SMP Srijayanegara Palembang on the Pythagorean Theorem material are still diverse in each component of Toulmin's argument, namely claims, data, warrants, backing, qualifiers, and rebuttals. Most students can write their claims or initial answer statements correctly. These findings indicate that most students can construct claims and provide data (evidence) when solving Pythagorean Theorem problems. This aligns with previous studies stating that students tend to be more capable of expressing answers and supporting them with basic information (AR et al., 2018; Resmi et al., 2021).

However, difficulties persist in more complex components, particularly warrants and backing, where students are required to explain the logical relationship between data and claims. Many students still cannot provide adequate backing and qualifiers to support their claims. This shows that students can form initial opinions on a math problem, but have not fully understood the underlying reasons and steps.This suggests that although students can arrive at answers, their underlying reasoning is not yet fully developed.

From the interview results, it is evident that some students tend to memorize the Pythagorean Theorem formula without understanding the relationship between the sides of a right triangle. As a result, their arguments are only procedural and lack logical support. Students who have a better understanding of concepts can present arguments more fully, accompanied by explanations of the reasons for using formulas and the relationships among triangle elements. Supporting components, such as theoretical explanations or relevant mathematical rules, appear in only a small percentage of students. Meanwhile, some qualifiers and rebuttals did not appear in the students' answers, suggesting that their ability to consider the limitations or other possibilities of a statement remained low.

These findings indicate that most students can construct claims and provide data (evidence) when solving Pythagorean Theorem problems. This aligns with previous studies stating that students tend to be more capable of expressing answers and supporting them with basic information (Ar et al., 2018; Resmi et al., 2021). However, difficulties persist in more complex components, particularly warrants and backing, where students are required to explain the logical relationship between data and claims. This suggests that although students can arrive at answers, their underlying reasoning is not yet fully developed.

This result is in line with the findings of Novia et al. (2017), who found that students' mathematical argumentation skills at the junior high school level are generally still limited to the ability to provide statements and data without strong reasons. Based on the overall analysis, students' argumentative skills are classified as high to medium. Influencing factors include a limited understanding of the Pythagorean Theorem and students' lack of familiarity with conveying mathematical reasons in a structured manner. Thus, the application of the Toulmin Argumentation Model can help teachers analyze the extent to which students can construct logical and complete arguments. This model can also serve as a reference in mathematics learning, so that students focus not only on the final result but also on the thinking process and the reasons behind solving problems.

In addition, the qualifier and rebuttal components appear infrequently. This finding supports previous research (Lubis & Lubis, 2024; Agustiningsih et al., 2021) showing that students are generally not accustomed to considering limitations, alternative perspectives, or counterarguments. From the perspective of the Toulmin Argumentation Model, these components represent higher-order argumentation skills, as they require students to evaluate the validity of their claims in different contexts and anticipate possible contradictions. The absence of these components indicates that students' argumentation is still dominated by linear thinking rather than reflective and evaluative reasoning.

Theoretically, the findings of this study reinforce the Toulmin Argumentation Model as an effective framework for analyzing the structure and quality of students' arguments. The model not only identifies whether students can provide answers, but also reveals the completeness of their reasoning through its six components (claim, data, warrant, backing, qualifier, and rebuttal). The imbalance found in this study—where claim and data are dominant, but warrant, backing, qualifier, and rebuttal are weaker—highlights that students' argumentation development is uneven and requires targeted instructional support.

Furthermore, these results imply that mathematics learning should not merely emphasize obtaining correct answers, but also focus on developing students' ability to justify, explain, and evaluate their reasoning. Instructional strategies that explicitly integrate argumentation practices, such as encouraging students to explain their reasoning, challenge peers' ideas, and consider alternative solutions, are essential to strengthen all components of argumentation.

CONCLUSION

Based on the research on the argumentation ability of grade VIII students in the Pythagorean Theorem material using the Toulmin Argumentation Model, it can be concluded that students' argumentation ability is generally in the high category. There were 23 students who participated in the study; 2 (8.7%) were categorized as low, 8 (34.8%) as moderate, and 13 (56.5%) as high.

While most students demonstrate relatively high argumentation ability in terms of claims and evidence, their ability to construct complete, well-structured arguments based on the Toulmin framework remains limited. Therefore, future learning should emphasize the development of deeper reasoning processes, particularly in the components of warrant, backing, qualifier, and rebuttal, to foster more comprehensive mathematical argumentation skills.

DECLARATIONS

Author Contribution : LMP: Conceptualization, Writing Original Paper, Data Collector; YH: Supervisor and Validation.

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