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The study examined students' mathematical reasoning skills in evaluating functions using GeoGebra in arithmetic sequence at senior high school

Tia Rahmawati[1], Indaryanti[2]
[1,2]Mathematics Education Department, Universitas Sriwijaya, Palembang, Indonesia
Correspondence: indaryanti@fkip.unsri.ac.id

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Abstract

Mathematical reasoning ability remains relatively low, particularly in arithmetic sequence material that requires skills in recognizing patterns and evaluating functions. Therefore, it is necessary to apply the Inquiry-Based Learning method, assisted by GeoGebra, in the learning process. This study aims to describe students' mathematical reasoning abilities through Inquiry-Based Learning assisted by GeoGebra in evaluating arithmetic sequence functions in Grade XI of SMA Negeri 8 Palembang. The research employed a descriptive method with 28 students from Class XI.6 as subjects. Data were collected through a written test consisting of three essay questions based on mathematical reasoning indicators to measure students' ability to evaluate arithmetic sequence functions. Data analysis was conducted descriptively based on the indicators of students' mathematical reasoning abilities. The results showed that the average mathematical reasoning ability of students was in the good category, with a percentage of 68.9%. The distribution of students' abilities consisted of 39.29% (11 students) in the excellent category, 17.86% (5 students) in the good category, 39.29% (11 students) in the fair category, and 3.57% (1 student) in the poor category. Research findings indicate that the use of GeoGebra in inquiry-based learning supports students in understanding arithmetic sequences, evaluating functions, and determining the n-th term in a more structured and logical manner.

Keywords: mathematical reasoning ability; evaluating functions; arithmetic sequence; inquiry-based learning; GeoGebra

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INTRODUCTION

Mathematical reasoning skills are a crucial competency in mathematics learning, as they facilitate logical, systematic, and critical thinking processes in problem-solving (Fitri et al., 2023). Mathematical reasoning plays a role in helping students develop targeted and reasonable steps to solve problems. Lubis & Hakim (2025) also state that mathematical reasoning skills are the basis for students to understand fundamental concepts and draw correct conclusions. Furthermore, Ulfiyati et al. (2025) explain that mathematical reasoning enables students to recognize patterns, construct arguments, and evaluate the truth of a mathematical statement or function. The importance of mathematical reasoning skills is also highlighted in research by Hadi et al. (2025), which states that mathematical reasoning is the foundation for other mathematical skills in various fields, enabling students to approach problems more critically and systematically. Therefore, mathematical reasoning is not only viewed as a procedural skill, but as a core process in understanding mathematics meaningfully (Alderton & Pratt, 2025).

However, various studies show that students' mathematical reasoning skills are still not optimally developed. Ramadhani et al. (2025) report that students continue to struggle with presenting mathematical arguments logically and coherently. Silalahi et al. (2024) found that students experience obstacles in performing deductive reasoning and evaluating the truth of a mathematical argument. Wulandari et al. (2025)demonstrated that the weak ability to relate mathematical concepts to real-world contexts also reflects a low level of students' mathematical reasoning. This was also mentioned by Sukirwan et al. (2018), who revealed that students still experience obstacles in reasoning in general, and most students' reasoning skills are at the imitative level. This condition was also seen in the initial observations at SMA Negeri 8 Palembang, where some students were unable to organize mathematical evidence or arguments systematically. This suggests that the students of SMA Negeri 8 Palembang possess relatively weak mathematical reasoning skills.

One of the mathematics topics that requires strong reasoning skills is arithmetic sequences, particularly in evaluating the n-th term function. Hidayat & Ihsan (2020) state that arithmetic sequences are related to contextual problems, such as financial calculations, thus requiring a deep understanding of the concept. Oktaviana & Aini (2021) explain that solving arithmetic sequence problems requires students' ability to recognize patterns and draw logical conclusions. However, Wau et al. (2022) found that students' mathematical reasoning skills in arithmetic sequences are still low. Fitria et al. (2023) also demonstrated that students continue to struggle with presenting mathematical statements in writing regarding arithmetic sequences and series.

To overcome the problem of low mathematical reasoning, a learning approach is needed that not only focuses on teachers but also encourages students to actively engage in the thinking process and build their own understanding (Safaat et al., 2025). Larsen & Misfeldt (2021) state that a learning environment that involves student exploration and understanding has a significant influence on improving students' mathematical reasoning skills. Therefore, inquiry-based learning is considered a practical approach for addressing issues related to mathematical reasoning skills. Gorat & Haryadi (2020) stated that inquiry-based learning can improve students' mathematical reasoning skills. Indah & Nuraeni (2021)demonstrate that inquiry-based learning offers students opportunities to construct knowledge through the investigative process. Waluyo & Bima (2023) emphasize that the use of inquiry-based learning tools supports the development of students' mathematical reasoning in a more structured manner. Research by Şen et al. (2021) revealed that students' mathematical reasoning abilities improved during learning using inquiry-based learning, where students developed and generalized operational strategies using mathematical expressions and drew conclusions based on their reasoning.

In addition to learning approaches, the use of technology also plays a role in supporting the development of students' mathematical reasoning. Sriyanta (2023) explains that digital technology enables learning to take place more dynamically and adaptively. Yustitia et al. (2024)demonstrate that learning technology facilitates students' understanding of concepts through interactive visualization and contextual learning experiences. One technology that has these characteristics is GeoGebra. Research conducted by Ulhanan et al. (2025) states that the use of GeoGebra supports the development of students' mathematical reasoning skills through visual exploration. Wulandari et al. (2020)demonstrate that GeoGebra-assisted teaching materials facilitate students' understanding of mathematical concepts in a more meaningful way. Hazira et al. (2024) report that GeoGebra applet-assisted learning encourages students to test and evaluate the mathematical ideas they have constructed during learning. A study by Negara et al. (2022)found that the use of GeoGebra can significantly improve mathematical reasoning skills. Furthermore, research by Khalil et al. (2017) states that a learning environment integrated with technology, such as GeoGebra, can improve mathematical thinking, including mathematical reasoning skills.

Several studies also demonstrate the effectiveness of GeoGebra and inquiry-based learning independently. Suciati et al. (2022) reported that learning using GeoGebra can support the improvement of students' mathematical reasoning skills. Hardiana et al. (2025)demonstrated that integrating GeoGebra into digital worksheets has a positive impact on students' mathematical reasoning. Laja (2020) stated that inquiry-based learning is efficacious in improving mathematical reasoning skills. Triatma et al. (2020) also showed that inquiry-based learning has a positive impact on students' mathematical reasoning skills. Studies on the use of GeoGebra in arithmetic sequence material are still relatively limited.

Based on this description, it can be seen that although GeoGebra and inquiry-based learning have been widely studied, there has not been much research describing students' mathematical reasoning abilities in the context of integrating the two, particularly in evaluating the n-th term of arithmetic sequences. Therefore, this study aims to describe students' mathematical reasoning abilities and explore how GeoGebra, in the context of inquiry-based learning, helps students evaluate arithmetic sequence functions.

METHODS

The type of research used is descriptive research because this study aims to provide an in-depth and informative description of the level of mathematical reasoning ability of high school students on arithmetic sequences in the context of GeoGebra-assisted inquiry-based learning, through the presentation of data in the form of a summary of centralization and distribution (Hikmawati, 2017; Sofiyana et al., 2022; The, 2024). This study focused on one main variable: the mathematical reasoning abilities of high school students in arithmetic sequences after participating in inquiry-based learning assisted by GeoGebra. The subjects of this study were 28 students of class XI.6 at SMA Negeri 8 Palembang in the odd semester of the 2025/2026 academic year. The research was conducted in three meetings: two meetings were used for learning through the inquiry-based learning model assisted by GeoGebra, and the third meeting was for a written test (posttest) to measure students' mathematical reasoning abilities. This research did not use a pretest because, based on the results of field surveys and brief interviews with mathematics teachers, it was found that students' mathematical reasoning skills in arithmetic sequences were still relatively low. Therefore, the research focused on describing students' mathematical reasoning skills after participating in the learning process. The research procedure consisted of three stages: the preparation stage, the implementation stage, and the data analysis stage. During the preparation stage, the researcher prepared the necessary research instruments and obtained the required administrative permits to conduct the research. During the implementation stage, the researcher employed GeoGebra-assisted inquiry-based learning and administered tests to assess mathematical reasoning ability. In the data analysis stage, the researcher processed the data obtained from the ability tests and categorized it according to mathematical reasoning ability. In terms of data collection techniques, this study employed a written test comprising three questions based on predetermined mathematical reasoning indicators. In terms of data analysis techniques, students' answers were scored according to the scoring guidelines. Then percentages were calculated to determine students' mathematical reasoning abilities based on the study by Ramadhanti & Marlina (2022) as presented in the following table:

Table 1. Criteria for Interpretation of Mathematical Reasoning Ability Scores

No	Percentage	Level of Mathematical Reasoning
1	0% - 20%	Very poor
2	21% - 40%	Poor
3	41% - 60%	Fair
4	61% - 80%	Good
5	81% - 100%	Very good

The results of these calculations provide a comprehensive overview of students' mathematical reasoning abilities in arithmetic sequences. In the final stage, researchers will draw conclusions based on the descriptive data obtained, thereby providing a general overview of high school students' mathematical reasoning abilities in arithmetic sequences after participating in inquiry-based learning assisted by GeoGebra.

RESULTS AND DISCUSSION

This study produced descriptive data analysis covering three main stages, namely planning, implementation, and analysis. In the initial stage, namely planning, the researcher carried out a series of preparations, including selecting the main issues to be the focus of the research, followed by determining the research location that was considered relevant to the focus of the study, then conducting observations at the school while also arranging for a research permit, followed by compiling and validating the research instruments, and determining the research subjects. This research was conducted at SMA Negeri 8 Palembang in the odd semester of the 2025/2026 academic year. During the observation at the school, discussions were held with one of the mathematics teachers to understand the conditions of the students and the class, as well as the suitability of the arithmetic sequence material, utilizing GeoGebra as a learning medium. The research subjects were also determined, namely class XI.6, which consisted of 28 students. After that, the researcher completed the administrative permits and determined the schedule for teaching and testing mathematical reasoning skills. The validated research instruments consisted of teaching modules, student worksheets, mathematical reasoning test questions, and assessment rubrics. The validation was carried out by two lecturers from the Mathematics Education Department of Sriwijaya University and one mathematics teacher from SMA Negeri 8 Palembang.

During the implementation stage, the research was conducted in three meetings, with two meetings focusing on learning and one meeting for researchers to perform mathematical reasoning ability tests. The first meeting focused on introducing GeoGebra in arithmetic sequences using inquiry-based learning methods with gadgets. In the core activity session, students worked on worksheets and were guided to recognize GeoGebra to visualize arithmetic sequence patterns in solving problems on the worksheets. The second meeting focused more on using GeoGebra to evaluate functions in arithmetic sequences. Students were directed to solve problems in the worksheets using GeoGebra to visualize the functions. At each meeting, students held group discussions and presented their findings.

At the beginning of the learning process, some students still struggled to answer questions on the worksheets, indicating that their mathematical reasoning skills were relatively low. However, as the learning process using inquiry-based learning, assisted by GeoGebra, progressed, an improvement in students' mathematical reasoning skills began to be observed. Students appeared to be better able to answer the questions on the worksheets. They began to actively ask questions to deepen and clarify their understanding of the material, enabling them to solve the problems correctly. In the second meeting, most students demonstrated an improved ability to answer the worksheet questions. Only a few students still experienced slight difficulties, particularly with questions that required the ability to construct proofs or reinforce solutions by linking concepts to graphs. This demonstrates that the application of inquiry-based learning, assisted by GeoGebra, can enhance students' mathematical reasoning skills through the process of independent exploration and discovery of concepts.

In the third meeting, researchers conducted a test to assess students' mathematical reasoning abilities after implementing the GeoGebra-assisted inquiry-based learning method. The test included three essay questions, each containing five sub-items, and students were given 100 minutes to complete it.

At the analysis stage, researchers scored the mathematical reasoning test using a rubric. Scores were converted to percentages. Results were grouped into five categories: "Very Good", "Good", "Fair", "Poor", and "Very Poor". The following table displays the average mathematical reasoning scores for evaluating functions in arithmetic sequences.

Table 2. Average Mathematical Reasoning Ability Score of Students

Percentage Interval	**Frequency**	**Percentage**	**Level of Mathematical Reasoning**
$0\% \leq score \leq 20\%$	0	0%	Very poor
$21\% \leq score \leq 40\%$	1	3,6%	Poor
$41\% \leq score \leq 60\%$	11	39,3%	Fair
$61\% \leq score \leq 80\%$	5	17,8%	Good
$81\% \leq score \leq 100\%$	11	39,3%	Very good
Sum	28	100%	
Average Score		68,9%	Good

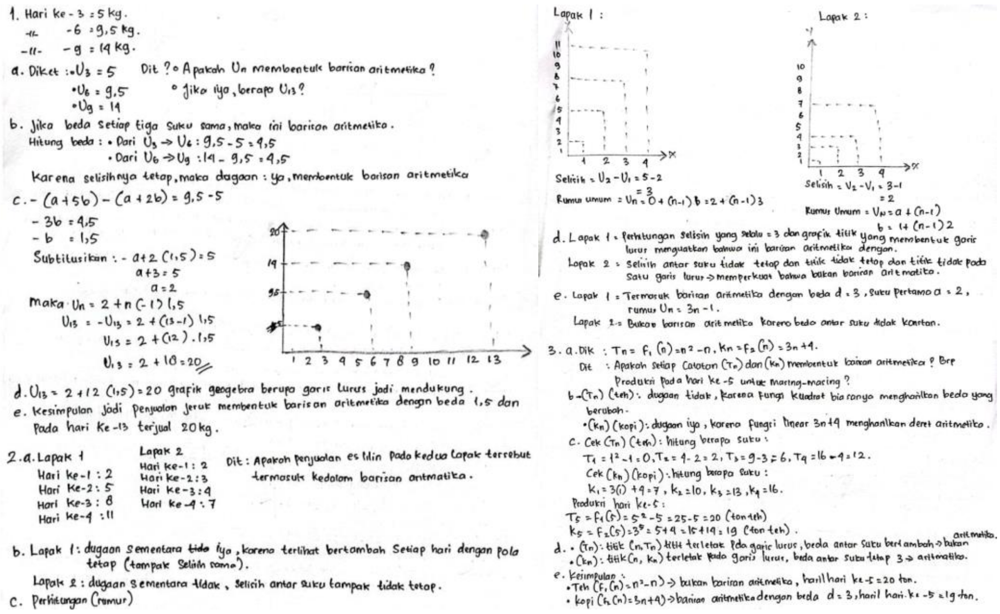
According to Table 2 above, the majority of students demonstrated mathematical reasoning skills in the "Very Good" and "Fair" categories, accounting for 39.3% of the students. Additionally, 17.8% of students fell into the "Good" category when evaluating functions in arithmetic sequences using GeoGebra. However, there are still 3.6% of students who fall into the "Poor" category, where students struggle to construct proofs or provide reasons to support solutions using GeoGebra when evaluating functions in arithmetic sequences. This is also supported by the average student score, as viewed through the mathematical reasoning ability indicator. These scores are presented in Table 3

Table 3. Average Mathematical Reasoning Ability Scores of Students Based on Indicators

Indicators of Mathematical Reasoning Ability	Percentage of Students
Expressing mathematical statements verbally, in writing, or through pictures or diagrams	88,99%
Formulating conjectures or hypotheses	65,18%
Skills in performing mathematical manipulations	72,02%
Ability to construct proofs or provide reasons supporting solutions	48,51%
Drawing logical conclusions based on available concepts or information	69,94%

Based on Table 3, it is known that students' ability to convey mathematical statements through writing and visuals is excellent, with a percentage of 88.99%. In addition, the level of student achievement in formulating hypotheses is 65.18%, in performing mathematical manipulations is 72.02%, and the level of achievement obtained by students in drawing logical conclusions from the available information is 69.94%. However, on the fourth indicator, namely the ability to compile evidence or provide reasons to support solutions, the lowest percentage of 48.51% was obtained. This suggests that although most students possess good basic mathematical reasoning skills, such as making statements, formulating assumptions, performing manipulations, and drawing conclusions, they still encounter difficulties when constructing logical arguments or providing evidence to support their solutions. This condition causes the average student score to fall into the "Good" category, even though some students have achieved the "Very Good" category.

To gain a deeper understanding of students' mathematical reasoning abilities on each indicator, an analysis of the students' test answer sheets was conducted. This analysis aimed to identify answer patterns, types of errors that appeared, and the types of solution strategies used by students on each question. The results of the analysis of students' answers, categorized by mathematical reasoning ability, are presented in Figure 1, with the "Very Good" category.



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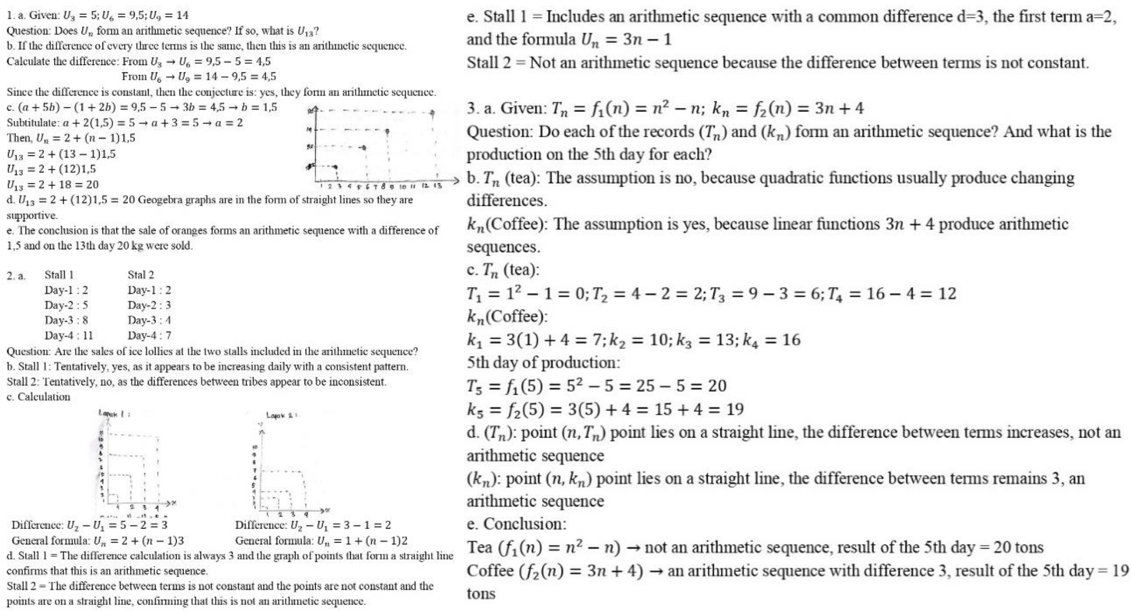


1. a. Given: $U_3 = 5; U_6 = 9{,}5; U_9 = 14$
Question: Does U_n form an arithmetic sequence? If so, what is U_{13}?
b. If the difference of every three terms is the same, then this is an arithmetic sequence.
Calculate the difference: From $U_3 \rightarrow U_6 = 9{,}5 - 5 = 4{,}5$
From $U_6 \rightarrow U_9 = 14 - 9{,}5 = 4{,}5$
Since the difference is constant, then the conjecture is: yes, they form an arithmetic sequence.
c. $(a + 5b) - (1 + 2b) = 9{,}5 - 5 \rightarrow 3b = 4{,}5 \rightarrow b = 1{,}5$
Subtitulate: $a + 2(1{,}5) = 5 \rightarrow a + 3 = 5 \rightarrow a = 2$
Then, $U_n = 2 + (n - 1)1{,}5$
$U_{13} = 2 + (13 - 1)1{,}5$
$U_{13} = 2 + (12)1{,}5$
$U_{13} = 2 + 18 = 20$
d. $U_{13} = 2 + (12)1{,}5 = 20$ Geogebra graphs are in the form of straight lines so they are supportive.
e. The conclusion is that the sale of oranges forms an arithmetic sequence with a difference of 1,5 and on the 13th day 20 kg were sold.

2. a.

Stall 1	Stal 2
Day-1 : 2	Day-1 : 2
Day-2 : 5	Day-2 : 3
Day-3 : 8	Day-3 : 4
Day-4 : 11	Day-4 : 7

Question: Are the sales of ice lollies at the two stalls included in the arithmetic sequence?
b. Stall 1: Tentatively, yes, as it appears to be increasing daily with a consistent pattern.
Stall 2: Tentatively, no, as the differences between tribes appear to be inconsistent.
c. Calculation
Difference: $U_2 - U_1 = 5 - 2 = 3$
General formula: $U_n = 2 + (n - 1)3$
Difference: $U_2 - U_1 = 3 - 1 = 2$
General formula: $U_n = 1 + (n - 1)2$
d. Stall 1 = The difference calculation is always 3 and the graph of points that form a straight line confirms that this is an arithmetic sequence.
Stall 2 = The difference between terms is not constant and the points are not constant and the points are on a straight line, confirming that this is not an arithmetic sequence.

e. Stall 1 = Includes an arithmetic sequence with a common difference d=3, the first term a=2, and the formula $U_n = 3n - 1$
Stall 2 = Not an arithmetic sequence because the difference between terms is not constant.

3. a. Given: $T_n = f_1(n) = n^2 - n$; $k_n = f_2(n) = 3n + 4$
Question: Do each of the records (T_n) and (k_n) form an arithmetic sequence? And what is the production on the 5th day for each?
b. T_n (tea): The assumption is no, because quadratic functions usually produce changing differences.
k_n(Coffee): The assumption is yes, because linear functions $3n + 4$ produce arithmetic sequences.
c. T_n (tea):
$T_1 = 1^2 - 1 = 0; T_2 = 4 - 2 = 2; T_3 = 9 - 3 = 6; T_4 = 16 - 4 = 12$
k_n(Coffee):
$k_1 = 3(1) + 4 = 7; k_2 = 10; k_3 = 13; k_4 = 16$
5th day of production:
$T_5 = f_1(5) = 5^2 - 5 = 25 - 5 = 20$
$k_5 = f_2(5) = 3(5) + 4 = 15 + 4 = 19$
d. (T_n): point (n, T_n) point lies on a straight line, the difference between terms increases, not an arithmetic sequence
(k_n): point (n, k_n) point lies on a straight line, the difference between terms remains 3, an arithmetic sequence
e. Conclusion:
Tea $(f_1(n) = n^2 - n) \rightarrow$ not an arithmetic sequence, result of the 5th day = 20 tons
Coffee $(f_2(n) = 3n + 4) \rightarrow$ an arithmetic sequence with difference 3, result of the 5th day = 19 tons

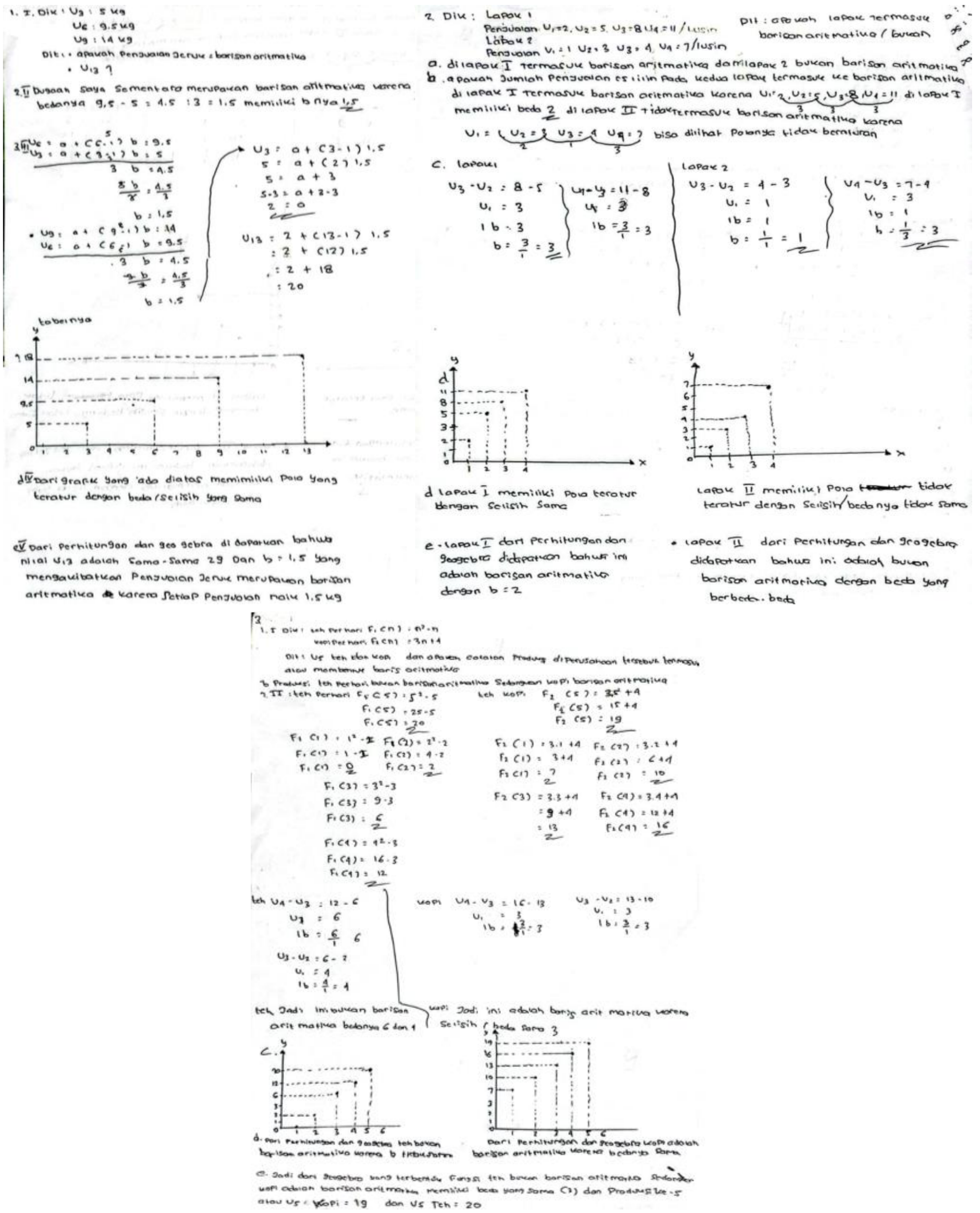
Figure 1. Student CK Numbers 1, 2, and 3's Answer with "Very Good" Category

Figure 1 shows the answers of the student with the initials CK to the three questions given. In the first question, CK was able to formulate information using appropriate mathematical language, construct hypotheses based on known information, perform calculations systematically, use GeoGebra correctly, and draw conclusions based on the results of calculations and graphs produced. However, CK still showed limitations in providing reasons to support the solution by linking the GeoGebra graph to the concept of arithmetic sequences. Therefore, in the first question, CK fulfilled four indicators of mathematical reasoning ability, while one indicator, namely providing supporting reasons, was not optimally fulfilled, resulting in a score of 18. In the second question, CK was able to interpret information using appropriate mathematical language, formulate hypotheses based on logical reasoning, and effectively utilize GeoGebra graphs.

However, the calculations were not entirely complete because they only determined the difference values without showing the similarity of the differences between terms.

Nevertheless, CK was able to provide supporting reasons for the solution by linking the GeoGebra graph to the concept of arithmetic sequences and drawing appropriate conclusions, thus obtaining a score of 19. In the third question, CK was able to formulate information using appropriate mathematical language, construct hypotheses with logical reasoning, and perform calculations correctly. However, CK did not utilize the GeoGebra graph in the solution and demonstrated limitations in providing reasons that linked the solution to the concept of arithmetic sequences. Nevertheless, CK was still able to conclude reasonably, thus obtaining a score of 17.

The results of the analysis of student answers based on mathematical reasoning ability, with a "Good" rating, are presented in Figure 2.



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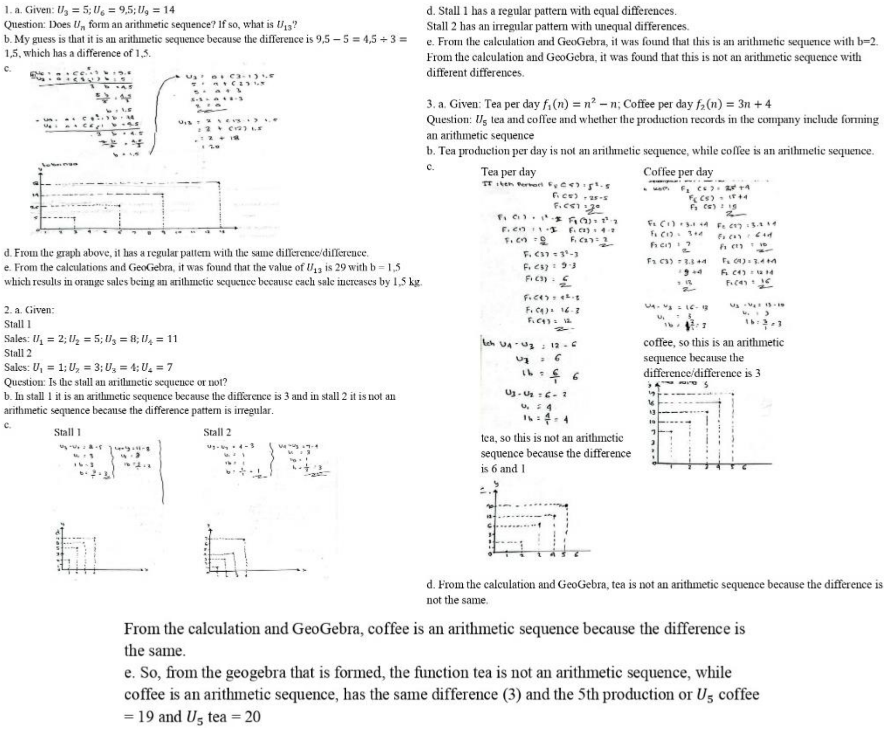
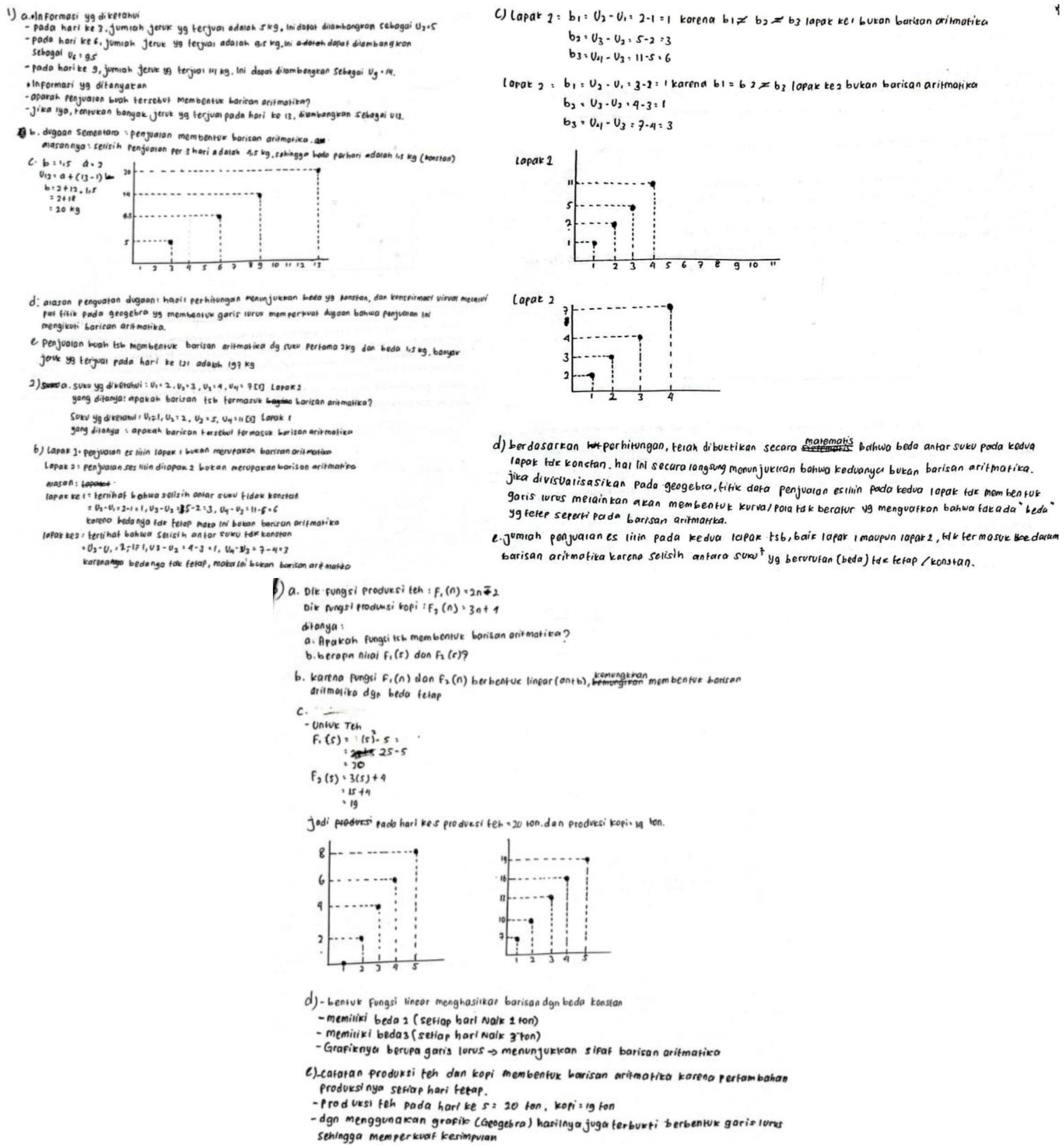


Figure 2. Student NUG Numbers 1, 2, and 3's Answer with "Good" Category

Figure 2 shows the answers of the student with the initials NUG to the three questions given. In the first question, NUG was able to write down the information from the question using correct and accurate mathematical language. NUG was also able to formulate a guess, but the reasons given were unclear and inaccurate. In mathematical manipulation, NUG performed the calculations correctly; however, errors were present in the GeoGebra graph used. Additionally, the reasoning provided to support the solution was not accurate because the relationship between the GeoGebra graph and the concept of arithmetic sequences was not explained clearly. NUG also drew conclusions that did not match the calculation results and the GeoGebra graph, resulting in a score of 11. In the second question, NUG was able to write down the information completely using appropriate mathematical language. NUG was also able to formulate assumptions accompanied by clear and logical reasoning. In mathematical manipulation, NUG performed calculations and used the GeoGebra graph well and accurately. However, the reasons given to support the solution were not adequately explained because they did not provide sufficient confirmation of the solution obtained.

Nevertheless, NUG was able to draw the correct conclusion based on the calculations and the GeoGebra graph, resulting in a score of 18. On the third question, NUG was able to write down the information using correct and accurate mathematical language and formulate a guess, even though it was not accompanied by reasoning. NUG performed the calculations correctly and used the GeoGebra graph appropriately. In providing reasons to support the solution, NUG effectively related the solution to the concept of arithmetic sequences and drew logical conclusions. Therefore, NUG received a score of 17.

Furthermore, the results of the analysis of student answers, categorized by mathematical reasoning ability, are depicted in Figure 3 for the category "Fair".



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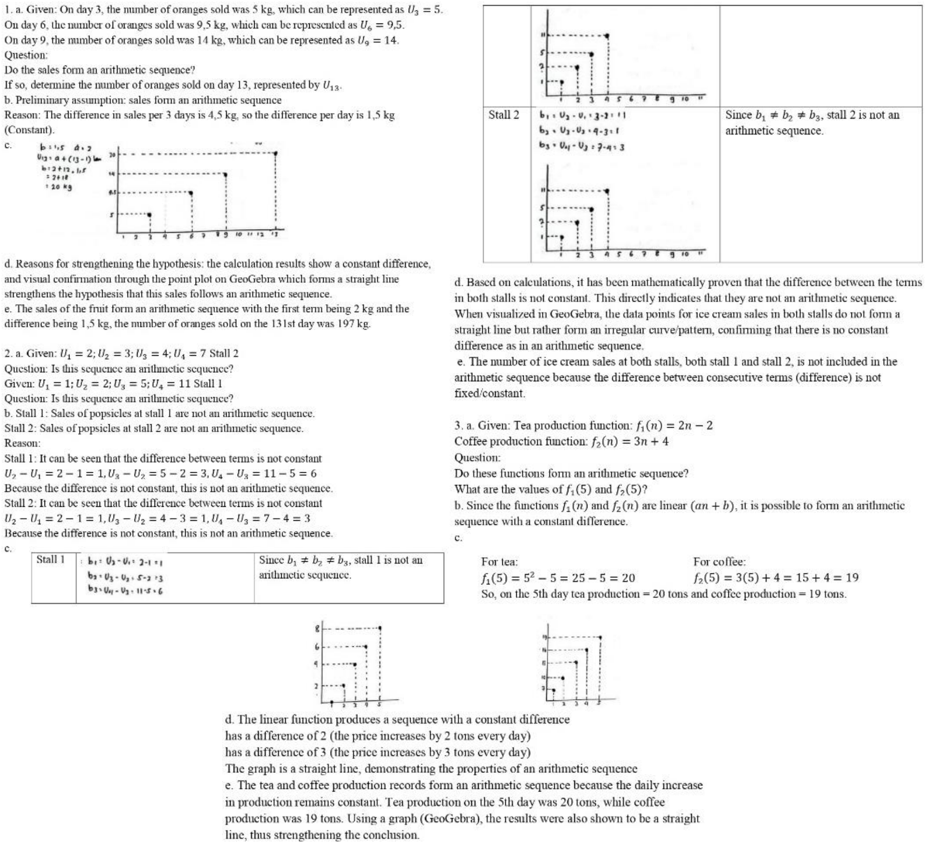

1. a. Given: On day 3, the number of oranges sold was 5 kg, which can be represented as $U_3 = 5$.
On day 6, the number of oranges sold was 9,5 kg, which can be represented as $U_6 = 9{,}5$.
On day 9, the number of oranges sold was 14 kg, which can be represented as $U_9 = 14$.
Question:
Do the sales form an arithmetic sequence?
If so, determine the number of oranges sold on day 13, represented by U_{13}.
b. Preliminary assumption: sales form an arithmetic sequence
Reason: The difference in sales per 3 days is 4,5 kg, so the difference per day is 1,5 kg (Constant).
c.

d. Reasons for strengthening the hypothesis: the calculation results show a constant difference, and visual confirmation through the point plot on GeoGebra which forms a straight line strengthens the hypothesis that this sales follows an arithmetic sequence.
e. The sales of the fruit form an arithmetic sequence with the first term being 2 kg and the difference being 1,5 kg, the number of oranges sold on the 131st day was 197 kg.

2. a. Given: $U_1 = 2; U_2 = 3; U_3 = 4; U_4 = 7$ Stall 2
Question: Is this sequence an arithmetic sequence?
Given: $U_1 = 1; U_2 = 2; U_3 = 5; U_4 = 11$ Stall 1
Question: Is this sequence an arithmetic sequence?
b. Stall 1: Sales of popsicles at stall 1 are not an arithmetic sequence.
Stall 2: Sales of popsicles at stall 2 are not an arithmetic sequence.
Reason:
Stall 1: It can be seen that the difference between terms is not constant
$U_2 - U_1 = 2 - 1 = 1, U_3 - U_2 = 5 - 2 = 3, U_4 - U_3 = 11 - 5 = 6$
Because the difference is not constant, this is not an arithmetic sequence.
Stall 2: It can be seen that the difference between terms is not constant
$U_2 - U_1 = 2 - 1 = 1, U_3 - U_2 = 4 - 3 = 1, U_4 - U_3 = 7 - 4 = 3$
Because the difference is not constant, this is not an arithmetic sequence.
c.

Stall 1	$b_1 = U_2 - U_1 = 2-1 = 1$ $b_2 = U_3 - U_2 = 5-2 = 3$ $b_3 = U_4 - U_3 = 11-5 = 6$	Since $b_1 \neq b_2 \neq b_3$, stall 1 is not an arithmetic sequence.
Stall 2	$b_1 = U_2 - U_1 = 3-2 = 1$ $b_2 = U_3 - U_2 = 4-3 = 1$ $b_3 = U_4 - U_3 = 7-4 = 3$	Since $b_1 \neq b_2 \neq b_3$, stall 2 is not an arithmetic sequence.

d. Based on calculations, it has been mathematically proven that the difference between the terms in both stalls is not constant. This directly indicates that they are not an arithmetic sequence. When visualized in GeoGebra, the data points for ice cream sales in both stalls do not form a straight line but rather form an irregular curve/pattern, confirming that there is no constant difference as in an arithmetic sequence.
e. The number of ice cream sales at both stalls, both stall 1 and stall 2, is not included in the arithmetic sequence because the difference between consecutive terms (difference) is not fixed/constant.

3. a. Given: Tea production function: $f_1(n) = 2n - 2$
Coffee production function: $f_2(n) = 3n + 4$
Question:
Do these functions form an arithmetic sequence?
What are the values of $f_1(5)$ and $f_2(5)$?
b. Since the functions $f_1(n)$ and $f_2(n)$ are linear $(an + b)$, it is possible to form an arithmetic sequence with a constant difference.
c.
For tea: $f_1(5) = 5^2 - 5 = 25 - 5 = 20$
For coffee: $f_2(5) = 3(5) + 4 = 15 + 4 = 19$
So, on the 5th day tea production = 20 tons and coffee production = 19 tons.

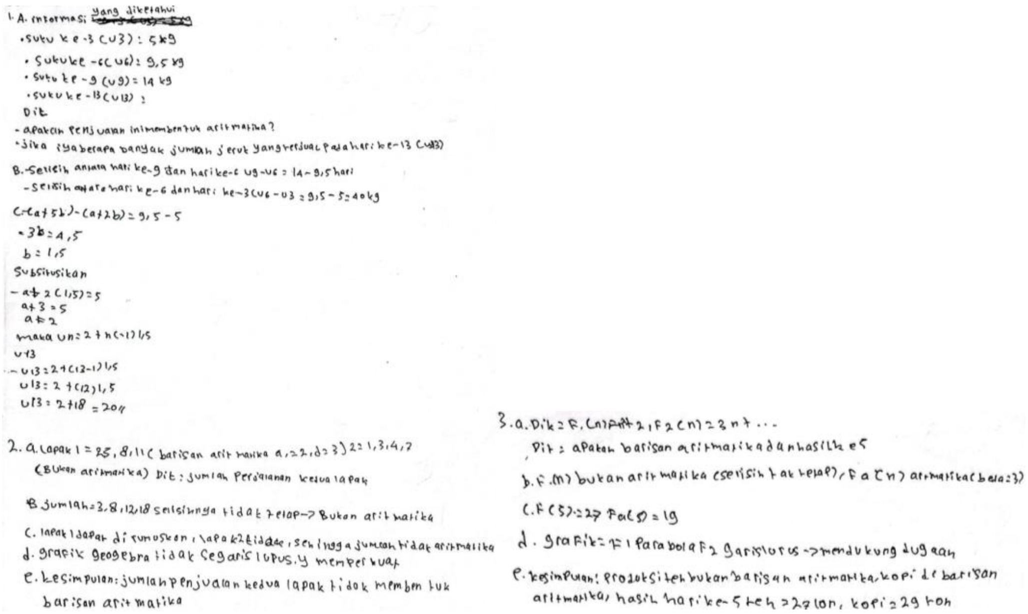
d. The linear function produces a sequence with a constant difference
has a difference of 2 (the price increases by 2 tons every day)
has a difference of 3 (the price increases by 3 tons every day)
The graph is a straight line, demonstrating the properties of an arithmetic sequence
e. The tea and coffee production records form an arithmetic sequence because the daily increase in production remains constant. Tea production on the 5th day was 20 tons, while coffee production was 19 tons. Using a graph (GeoGebra), the results were also shown to be a straight line, thus strengthening the conclusion.

Figure 3. Student KFAF Number 1, 2, and 3's Answer with "Fair" Category

Figure 3 shows the answers of the student with the initials KFAF to the three questions given. In the first question, KFAF was able to write down the information from the question using good and accurate mathematical language and provided assumptions accompanied by logical and appropriate reasons. In mathematical manipulation, the calculations written down were not systematic, but the use of GeoGebra graphs was done correctly. KFAF was also able to provide reasons supporting the solution by accurately connecting the concept of arithmetic sequences with the GeoGebra graph. However, the conclusion written was not in accordance with the solution, so KFAF received a score of 13. In the second question, KFAF wrote down the information from the question using mathematical language, but there were still many errors. The guesses given were not accurate, and the accompanying reasons were not appropriate. In mathematical manipulation, KFAF used incorrect information, resulting in incorrect calculations and GeoGebra graphs.

Nevertheless, KFAF was still able to relate arithmetic concepts in providing supporting reasons. However, KFAF was unable to draw correct and appropriate conclusions, resulting in a score of 8. In the third question, KFAF was able to write down the information from the question correctly, although there were still some errors. The provided guesses were inaccurate and lacked supporting reasons. In mathematical manipulation, KFAF was able to perform calculations and use GeoGebra correctly, although the calculation results still had many shortcomings. In the stage of providing supporting reasons, KFAF was not yet able to accurately connect arithmetic concepts with GeoGebra graphs. Additionally, the conclusions written were also inappropriate, resulting in a score of 8 for KFAF.

The following are the results of the analysis of student answers, categorized by mathematical reasoning ability, with the category "Poor" depicted in Figure 4.



English version

1. a. Given: 3rd term (U_3) = 5 kg
6th term (U_6) = 9,5 kg
9th term (U_9) = 14 kg
Question: Do these sales form an arithmetic sequence?
If so, how many oranges were sold on the 13th day (U_{13})?
b. difference between day 9 and day 6 ($U_9 - U_6 = 14 - 9{,}5\ days$)
difference between day 6 and day 3 ($U_6 - U_3 = 9{,}5 - 5 = 40\ kg$)
c. $(a + 5b) - (a + 2b) = 9{,}5 - 5$
$3b = 4{,}5$
$b = 1{,}5$
Substitute
$a + 2(1{,}5) = 5$
$a + 3 = 5$
$a = 2$
Then, $U_n = 2 + (n - 1)1{,}5$
$U_{13} = 2 + (13 - 1)1{,}5$
$U_{13} = 2 + (12)1{,}5$
$U_{13} = 2 + 18 = 20$

2.a. Stall 1 = 2, 5, 8, 11 (arithmetic sequence a = 2, d = 3)
Stall 2 = 1, 3, 4, 7 (not an arithmetic sequence)
Question:
Total number of trips between the two stalls
b. Total = 3, 8, 12, 18, the difference is not constant, not an arithmetic sequence.
c. Stall 1 can be formulated, but stall 2 cannot, so the total is not arithmetic.
d. The GeoGebra graph is not a straight line, which supports this.
e. Conclusion: The total sales of the two stalls do not form an arithmetic sequence.

3. a. Given: $f_1(n) = n^2 + 2$, $f_2(n) = 3n + \cdots$
Question: Is the sequence arithmetic and the 5th product?
b. $f_1(n)$ not an arithmetic sequence (the difference is not constant), $f_2(n)$ arithmetic (the difference = 3)
c. $f_1(5) = 27$, $f_2(5) = 19$
d. graph: f_1 is a parabola, f_2 is a straight-line, supporting the hypothesis
e. Conclusion: tea production is not an arithmetic sequence, coffee is an arithmetic sequence, the yield on day 5 is 27 tons of tea, and coffee is 29 tons.

Figure 4. Student YP Number 1, 2, and 3's Answer with "Poor" Category

Figure 4 shows the answers of the student with the initials YP to the three questions given. In the first question, YP was able to write down the information from the question correctly and accurately. However, the guess provided was incorrect and lacked a justification. In mathematical manipulation, YP performed the calculations well and completely, but did not display a graph as a form of using GeoGebra. Additionally, YP did not provide reasons to support the solution or draw conclusions, resulting in a score of 8. In the second question, YP was able to write down the information from the question, although there were still some shortcomings. The formulated hypothesis was incorrect and lacked supporting reasons. In mathematical manipulation, YP made incorrect calculations and did not use the GeoGebra graph. YP was also unable to provide reasons linking the solution to the concept of arithmetic sequences and draw appropriate conclusions, thus obtaining a score of 7. In the third question, YP was able to accurately record the information from the question. The formulated hypothesis was correct, but the reasons provided were not clear. In mathematical manipulation, YP made incorrect calculations and did not use GeoGebra graphs. Additionally, YP was unable to provide reasons linking the solution to the concept of arithmetic sequences and draw appropriate conclusions, resulting in a score of 9.

Based on the test data analysis of three questions that consider variations in students' mathematical reasoning abilities, a picture of the differences in achievement characteristics within each category was obtained. Students in the "Very Good" ability category generally met all mathematical reasoning indicators, although in some cases, there were still limitations in presenting strong reasons to support the solutions provided. Meanwhile, students in the "Good" category also generally met all indicators, but still experienced difficulties in formulating assumptions and providing supporting reasons, indicating that their reflective thinking skills were not yet fully developed. These findings align with the results of Mahmud & Drus (2023) research, which suggests that students with high reasoning abilities tend to solve problems procedurally correctly, but this is not always accompanied by in-depth reasoning.

In the "Fair" category, students still made mistakes in formulating assumptions and constructing supporting arguments, and there were limitations in mathematical manipulation. This indicates that students in this category were not yet able to integrate visual and symbolic representations optimally. Students in the "Poor" category experience difficulties in almost all indicators of mathematical reasoning, from formulating assumptions and mathematical manipulation to providing reasons and concluding. These findings align with previous research conducted by Ünal et al. (2023), which suggests that students frequently make mistakes in formulating assumptions and performing symbolic manipulation, often providing weak reasons, indicating a lack of integration between visual and symbolic representations. The same finding was reported in a study by Khansa et al. (2020), which noted that some students continue to experience difficulties with arithmetic operations. Thus, the differences in achievement between categories indicate variations in the level of conceptual internalization and reasoning ability among students, which is an essential finding in describing the mathematical reasoning profile of students in GeoGebra-assisted inquiry-based learning.

This indicates that some students continue to struggle with formulating guesses and providing evidence or reasons to support solutions by linking mathematical concepts, particularly arithmetic sequences, which are indicators of students' mathematical reasoning abilities. This aligns with the research by Vebrian et al. (2021), which suggests that students experience difficulties in mathematical reasoning skills because they are not accustomed to questions that require high-level reasoning and lack mastery of mathematical concepts. This is also clarified by the research of Sumartini & Utami (2023), which explains that the factors causing students' mathematical reasoning skills to be lacking are that students do not fully understand the questions, do not understand the mathematical concepts used in the questions, are careless in their work, are not accustomed to questions that require mathematical reasoning skills, and lack interest in working on the questions. Although the overall level of students' mathematical reasoning ability falls into the "Good" category, several indicators have not been fully mastered by all students, particularly in formulating assumptions and providing logical supporting reasons. Therefore, learning strategies are needed that can foster mathematical reasoning skills and accustom students to solving mathematical reasoning problems in depth.

CONCLUSION

Based on the study's results, students' mathematical reasoning skills in evaluating arithmetic sequences through inquiry-based learning, assisted by GeoGebra, were generally categorized as "Good." The findings show that students are better able to express mathematical statements, followed by performing mathematical manipulations, then drawing conclusions, formulating hypotheses, and gathering supporting evidence or arguments. This demonstrates that the use of GeoGebra in inquiry-based learning facilitates the visualization and exploration of concepts, thereby enabling students to understand patterns and relationships in arithmetic sequences in a more structured manner. This study suggests that teachers and students should make greater use of GeoGebra to strengthen mathematical reasoning. Meanwhile, other researchers can continue their studies on different learning materials or models to enrich the research results, particularly in improving mathematical reasoning skills.

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