

Students' analogical reasoning and creative thinking across mathematical ability levels

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Abstract

This study examines students' analogical reasoning and creative thinking in solving open-ended mathematical reflection problems across different levels of mathematical ability. Using a qualitative descriptive approach, data were collected from ninth-grade students through written tasks and interviews, with three participants representing high, medium, and low ability levels. The analysis focused on the stages of analogical reasoning: structuring, mapping, applying, and verifying based on Ruppert's model. The findings show apparent differences in students' reasoning processes. High-ability students demonstrated complete and systematic analogical reasoning across all stages and were able to justify their solutions logically. Medium-ability students demonstrated a partial understanding and relied mainly on procedural strategies, whereas low-ability students relied on intuitive visualization with limited mathematical justification. These differences suggest that creative thinking plays a crucial role in enhancing the completeness of analogical reasoning. The novelty of this study lies in its detailed analysis of how students with different mathematical ability levels progress through each stage of analogical reasoning when solving open-ended reflection problems. The results suggest that integrating analogical reasoning tasks into open-ended mathematics instruction can strengthen students' conceptual understanding and support the development of creative thinking.

Keywords: analogical reasoning; creative thinking; geometry reflection; mathematics education; open-ended problem.

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INTRODUCTION

Mathematical reasoning is a fundamental objective of mathematics education, as emphasized by the National Council of Mathematics (NCTM, 2020), because it enables students to connect concepts, recognize patterns, and justify conclusions logically (Payadnya, 2019; Salsabila et al., 2023; Widiartana, 2018). The Programme for International Student Assessment (PISA) 2022 reported that Indonesia ranked 71st out of 81 participating countries, suggesting persistent challenges in higher-order reasoning beyond procedural fluency (OECD, 2023). Such a gap indicates that many Indonesian students still struggle not only with procedural fluency but also with higher-order reasoning, suggesting the need to strengthen instructional practices that promote conceptual understanding and reasoning (Marasabessy, 2021; Napitupulu & Suryadi,

2016). Reasoning represents a fundamental characteristic of mathematics that cannot be separated from understanding and solving mathematical problems (Angraini et al., 2023; Kosyvas, 2016). Therefore, introducing and reinforcing reasoning in mathematics education is not merely necessary but essential for developing students' critical, systematic, and reflective thinking (Medová et al., 2020). In this regard, analogy serves as a cognitive bridge that helps students form deeper conceptual connections by relating new information to prior knowledge (Richland et al., 2004). Bartha (2013) defines analogy as a form of reasoning based on recognizing explicit similarities between two systems to support inferential processes. Analogical reasoning, therefore, refers to the cognitive process of mapping relationships between a source problem (a familiar situation) and a target problem (a novel situation) based on structural similarities (Gentner & Hoyos, 2017; Richland et al., 2004; Goswami, 2020). Through this process, students can transfer knowledge from known contexts to new ones by means of structure mapping (Gentner, 1983; Ruppert, 2013; Wijaya, 2018).

Ruppert (2013) proposed a four-stage model of analogical reasoning consisting of structuring, mapping, applying, and verifying. This model provides a comprehensive framework to analyze how students build, compare, and validate relational correspondences between mathematical concepts. It is particularly effective for studying students' reasoning in open-ended mathematical problem solving, as it captures the dynamic and iterative nature of cognitive processes involved in analogy. Several studies have confirmed that analogical reasoning supports the development of mathematical flexibility and creativity (Brookman-Byrne et al., 2019; Tzuriel, 2024). The open-ended approach is considered one of the most effective strategies to promote reasoning and creativity, especially in geometry topics such as reflection. Open-ended reflection problems encourage students to explore multiple solution paths and to understand reflections by identifying analogical relationships between objects and their images. This process stimulates creative thinking by generating original and flexible ideas (Rahmawati et al., 2022; Shim, 2023).

Nevertheless, reflection remains one of the most challenging topics for many students. Amalia (2023) reported that 69.56% of students at SMP Negeri 2 Kuala, Aceh, encountered difficulties in solving reflection problems. Similarly, a study by Maulani and Setiawan (2021) at SMK Negeri 1 Cimahi found that the highest error rate (57%) occurred

in the reflection subtopic of geometric transformations. These findings suggest that while reflection is a crucial component of the Grade IX geometry curriculum, it remains a persistent source of misunderstanding among students, thereby warranting further investigation.

Preliminary classroom observations conducted in several junior high schools in Surabaya revealed that many students still perceive reflection merely as a mechanical procedure of flipping a point over an axis without understanding the underlying relationships between distance, direction, and image position. When presented with open-ended reflection tasks, approximately 72% of students were able to determine the direction of the reflected point but failed to provide a logical mathematical justification. This phenomenon highlights a gap between conceptual understanding and analogical reasoning, both of which directly influence students' creative mathematical thinking. Moreover, most existing studies have treated analogy as a single cognitive event rather than as a sequence of interactive stages intertwined with the creative process. Empirical evidence also remains limited regarding how students with different levels of mathematical ability (high, medium, and low) navigate each stage of analogical reasoning as described by Ruppert (2013).

Based on these considerations, this study aims to identify the stages of analogical reasoning demonstrated by junior high school students in solving open-ended reflection problems and to analyze the patterns of mathematical creative thinking, including fluency, flexibility, and originality, that emerge at each stage. By doing so, this research aims to provide deeper insights into the cognitive mechanisms that link analogical reasoning and mathematical creativity, thereby contributing to the development of more effective instructional strategies for fostering creative and conceptual understanding in mathematics learning.

METHODS

This study employed a qualitative descriptive approach to examine students' analogical reasoning processes in solving open-ended mathematical reflection problems. The main focus was to describe the stages of analogical reasoning based on Ruppert's (2013) model, which consists of structuring, mapping, applying, and verifying, and to

identify reasoning patterns that appeared when students solved open-ended mathematical reflection problems.

The participants were 18 ninth-grade students from the 2025/2026 academic year. All students completed a Mathematical Ability Test (MAT), which consisted of 0 items combining multiple-choice and open-ended questions designed to assess conceptual understanding, logical reasoning, and problem-solving skills in geometry and algebra. Based on their MAT scores, students were categorized into three groups: high, medium, and low mathematical ability. From these groups, three students were selected purposively to represent each ability level. This selection was considered sufficient for in-depth qualitative analysis, as the study aimed to explore reasoning processes rather than to generalize statistically. Three instruments were used: (1) the Mathematical Ability Test (MAT) to classify students' ability levels, (2) an Open-Ended Analogical Reasoning Task on reflection concepts to explore how students used analogies in solving problems, and (3) a Semi-Structured Interview Protocol to investigate their reasoning processes in greater depth. The Mathematical Ability Test is used to classify students into ability levels. Open-Ended Analogical Reasoning Tasks, consisting of two reflection problems that required students to explain their reasoning and justify solutions. Semi-structured interviews were conducted to clarify students' written responses and to delve deeper into their reasoning processes. Data were collected through written tests, interviews, and documentation, all of which were triangulated for validation.

Ethical approval was obtained from the school authorities prior to the commencement of data collection. Informed consent was secured from students and their guardians. Participants' identities were anonymized using initials to ensure confidentiality, and all data were used solely for research purposes. Additionally, participant characteristics, ability levels, and the analysis framework are presented in Table 1.

Table 1. Participant Characteristics, Ability Levels, and Analysis Framework

Participant	Gender	Ability Level	Score Range	Reasoning Characteristics	Creative Thinking Indicators (Silver, 1997)	Stages of Analogical Reasoning (Ruppert, 2013)
NF	Female	Low	$x \leq 33$	Relied on procedural intuition and lacked conceptual connection	Low fluency and flexibility with limited elaboration	Reached the structuring stage only
AS	Female	Medium	$33 < x < 67$	Recognized relationships but inconsistent in verification	Moderate fluency and adaptability with conventional responses	Completed the four stages procedurally
MA	Male	High	$x \geq 67$	Logical and reflective with strong conceptual generalization	High fluency, flexibility, and originality	Completed the four stages systematically

Data were analyzed using the framework of Miles, Huberman, and Saldaña (2019), which includes data condensation, data display, and conclusion drawing. Students' written responses and interview transcripts were analyzed to identify indicators of the structuring, mapping, applying, and verifying stages. Verification was identified through students' justification, checking procedures, and consistency between written and verbal explanations. Triangulation between written work and interview data was used to enhance the credibility of the findings. The integration of these data sources provided a comprehensive understanding of students' analogical reasoning according to their mathematical ability levels.

To assess students' creative thinking, four observable indicators were used: fluency, flexibility, originality, and elaboration. These indicators were applied alongside the four stages of analogical reasoning, namely structuring, mapping, applying, and verifying, to examine the relationship between creative thinking and the quality of reasoning. Triangulation among written, verbal, and observational data ensured the validity and credibility of the findings.

RESULTS AND DISCUSSION

Students' mathematical abilities were identified through the Mathematical Ability Test (MAT), which measured conceptual understanding, logical reasoning, and problem-solving skills. Based on the results, participants were grouped into three levels of mathematical ability: high, medium, and low. Three representative students were selected: MA (high), AS (medium), and NF (low). Analysis revealed that each participant exhibited distinct, measurable profiles in creative thinking and analogical reasoning when solving open-ended reflection problems, as summarized in the following tables. Each participant demonstrated different patterns of innovative thinking and analogical reasoning when solving open-ended reflection problems. Table 1 summarizes their main characteristics, showing that NF relied heavily on intuition, AS demonstrated partial conceptual understanding, and MA exhibited systematic and logical reasoning. Furthermore, participant characteristics are presented in Table 2.

Table 2. Participant Characteristics

Participant	Characteristics
NF (Low Ability)	Relied heavily on intuition and visualization, able to depict the reflection direction correctly, but struggled to relate the object and image mathematically. Her reasoning process remained at the initial structuring stage without progressing to verification.
AS (Medium Ability)	Displayed a moderate understanding of reflection concepts and could identify key ideas, though inconsistencies appeared in verifying results. She related examples to new problems but occasionally lacked accuracy in mapping object–image relationships.
MA (High Ability)	Demonstrated reflective, systematic, and logical mathematical thinking. He generated multiple solution strategies, justified each reasoning step, and successfully connected mirror reflection concepts with coordinate transformations.

Following the MAT, all three participants solved two open-ended reflection problems designed to trace their analogical reasoning according to Ruppert's stages: structuring, mapping, applying, and verifying. Analysis of their written responses and interviews revealed apparent differences across ability levels.

1. NF (Low Mathematical Ability)

NF's written work (see Figure 1) demonstrated that she recognized the general direction of the image but struggled to apply mathematical principles to connect position, distance, and orientation. She intuitively understood that the image appeared in the

opposite direction and was symmetrically located relative to the mirror, but her reasoning lacked coordinate representation. During the interview, she stated, "*I imagine a mirror, like when we look at ourselves. The reflection is in the opposite direction,*" indicating reliance on perceptual rather than mathematical reasoning. When asked to determine coordinates, she admitted that she did not know how to express them symbolically, demonstrating awareness of symmetry without resorting to abstraction.

NF did not successfully establish a correspondence between the object and image during the mapping stage. She relied on memorized statements rather than conceptual understanding, only mentioning that "*the distances are the same and the image is inverted.*" The application stage appeared incomplete, and no verification occurred. Overall, her reasoning was limited to the structuring stage, supported by imagination rather than analytical understanding.

The image produced by NF appeared reversed, with the image facing opposite the observer and located precisely behind the mirror at an equal distance from it. The image was symmetrically positioned relative to the mirror plane and of the same size as the object. However, it experienced a perceptual reversal of left and right rather than a front-to-back inversion. For instance, a point reflected across the line $y = 0$ produced an image below the x -axis, while reflection across $y = x$ resulted in interchanged coordinates. This demonstrated NF's awareness of visual symmetry but not symbolic or quantitative understanding. The NF answer is presented in Figure 1.

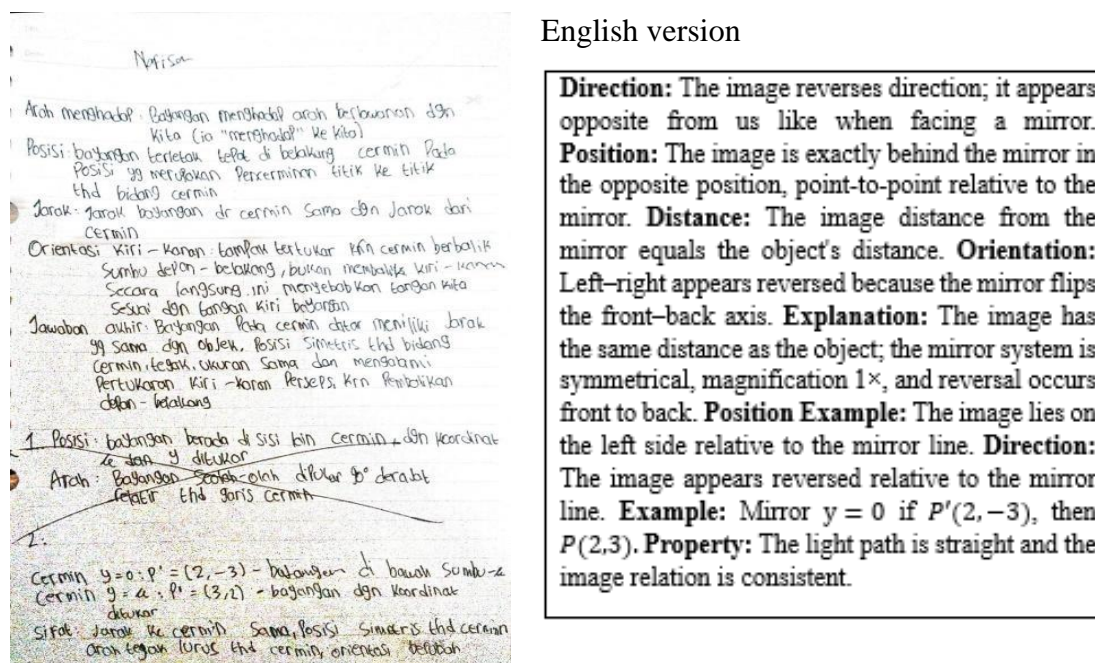


Figure 1. NF Answer Results

From the perspective of creative thinking, NF demonstrated low fluency, flexibility, and elaboration. Her responses revealed a single, straightforward approach without exploring alternatives. Her reasoning was rigid and procedural, showing minimal originality or expansion of ideas.

2. AS (Medium Mathematical Ability)

AS's written work (see Figure 2) demonstrated stronger conceptual understanding. She correctly explained that when the right hand is raised, the reflection appears to raise the left, showing awareness of directional inversion. At the structuring stage, she identified that the image's distance from the mirror was equal to that of the object. During the interview, she stated, "I imagine the mirror along the x -axis or y -axis and see how the point moves. For the x -axis, the sign of the y -coordinate changes." This shows her shift from visual to symbolic reasoning.

At the mapping stage, AS established coordinate correspondences correctly, such as transforming $P(2, -3)$ into $P'(2, 3)$ when reflected across the x -axis. During the applying stage, she determined image positions procedurally and verified the equality of distances. In the verifying stage, she stated, "I checked that the distances were equal and the positions symmetrical," indicating reflective awareness.

AS's reasoning reflected all four stages of analogical reasoning, although it was still procedural. Her image interpretation described that when an observer raised the right

hand, the image appeared to raise the left hand. The image was upright, positioned behind the mirror at an equal distance, and perpendicular to the wall. Reflections with respect to the x -axis resulted in equal distances of point P and its image P' from the axis, with the x -coordinate changing sign. The image was virtual, upright, and equal in size, showing direction reversal and spatial symmetry. The AS answer is presented in Figure 2.

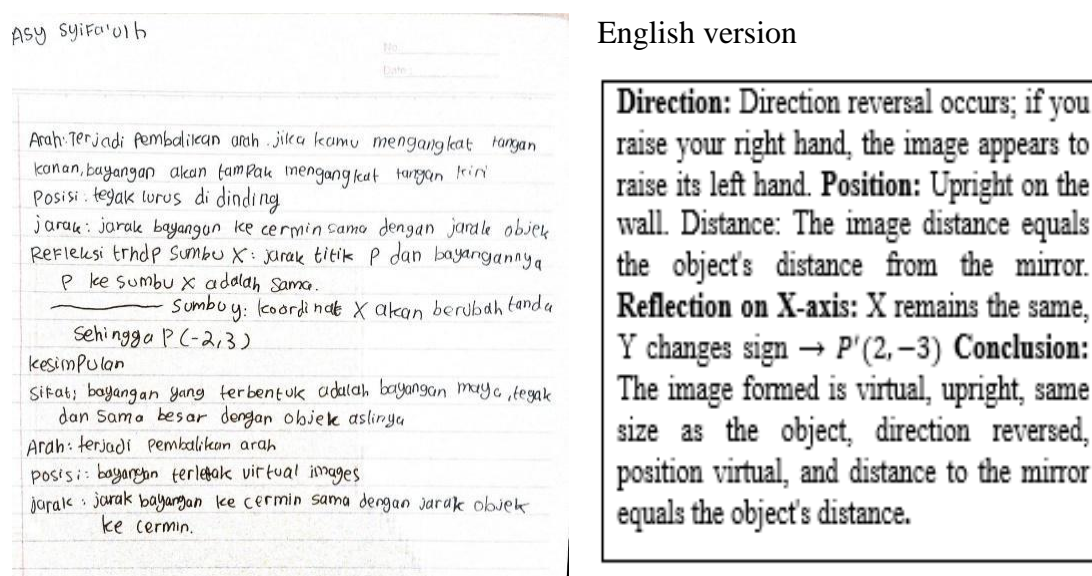


Figure 2. AS Answer Results

AS demonstrated moderate creative thinking, with good fluency and flexibility. She could shift between algebraic and diagrammatic representations when facing difficulties, but her originality and elaboration remained conventional.

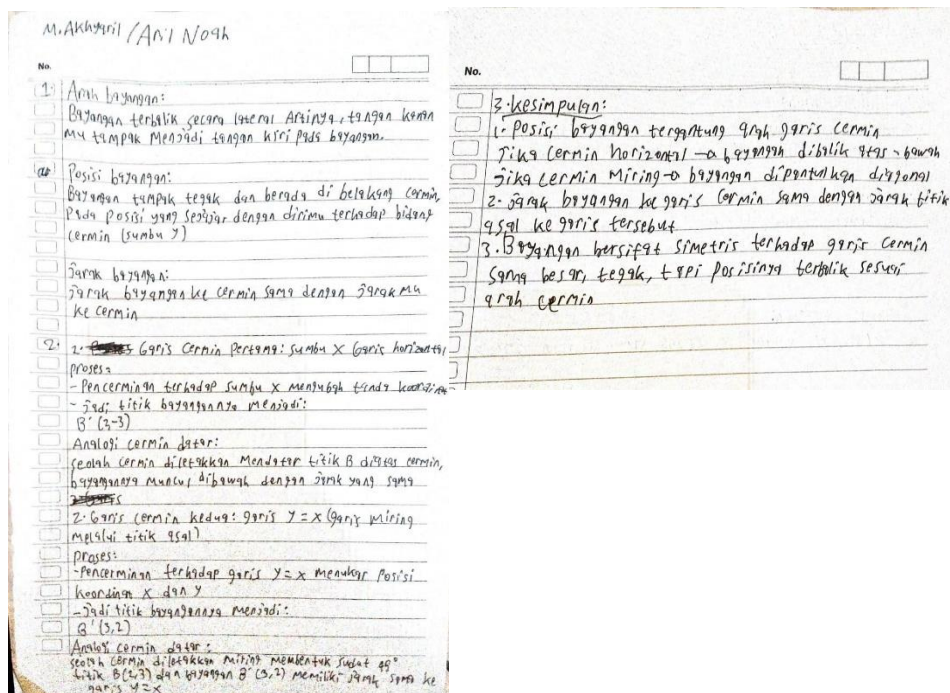
3. MA (High Mathematical Ability)

MA's written work (see Figure 3) demonstrated the most complete and systematic reasoning. He correctly described direction, position, and distance, deriving precise reflection rules for different axes. At the structuring stage, MA began his reasoning with conceptual analysis. He explained, "I first identify the reflection line, whether it is the x -axis, y -axis, or $y = x$, and then apply the corresponding rule."

At the mapping stage, MA established accurate correspondences, for instance, transforming $P(2, 3)$ into $P'(2, -3)$ across the x -axis and $P'(3, 2)$ across the line $y = x$. During the applying stage, he executed transformations logically using algebraic substitution and geometric visualization. He explained, "I checked whether the image was equidistant from the mirror and compared it to how light reflects in physics." This

indicated cross-domain analogical reasoning. In the verifying stage, he confirmed his results through sketches and recalculations, showing strong metacognitive control.

MA described the image as appearing reversed, upright, and symmetrically positioned behind the mirror. A flat mirror along the x-axis produced reflections with coordinates $(x, -y)$, while reflection across $y = x$ interchanged coordinates (y, x) . The image's position depended on the mirror's orientation: when the mirror was horizontal, the image appeared below the axis; when the mirror was oblique, the reflection followed the mirror's slope. The image remained symmetrical, equal in size, upright, and perceptually reversed. The MA answer is presented in Figure 3.



English version

Image Direction: The image reverses direction; raising the right hand appears as the left in the mirror. **Position:** Upright, behind the mirror, symmetrical to the object. **Distance:** Distance of image to mirror equals object distance. **With Mirror Line $y = x$:** Reflection swaps x and y values. Example: $(2,3) \rightarrow (3,2)$ **Mirror Rule:** A diagonal mirror causes coordinate swapping. **Conclusion:** (1) Image position depends on mirror orientation: horizontal \rightarrow reflected up/down; diagonal \rightarrow coordinates change. (2) Distance of image to mirror equals object's distance. (3) Image is symmetrical, same size, upright, but position follows mirror direction.

Figure 3. MA's Answer Result

From a creative thinking perspective, MA exhibited high fluency, flexibility, originality, and elaboration. He generated multiple solution strategies, connected mathematical reflection with optical concepts, and provided detailed reasoning and verification.

4. Comparative Analysis of Analogical Reasoning

A comparative analysis of the stages of analogical reasoning is presented in Table 3.

Table 3. Comparative Analysis of Analogical Reasoning Stages

Analogical Reasoning Stage	NF (Low)	AS (Medium)	MA (High)
Structuring	Understood the general concept of opposite direction, but lacked a mathematical model.	Identified mirror axis and image direction.	Defined reflection rules clearly with reference to direction, position, and image properties.
Mapping	Unable to connect the object and image mathematically.	Linked sign changes in coordinates to mirror positions.	Mapped object and image accurately, including oblique reflections.
Applying	Noted equal distances but without computation.	Applied transformations correctly but unsystematically.	Executed transformations logically and consistently.
Verifying	Did not verify results.	Drew simple conclusions about an upright and congruent image.	Verified results through recalculation and sketching.

Overall, these findings reinforce the argument of English (2004) and Gentner and Hoyos (2017) that analogical reasoning bridges conceptual understanding and mathematical application. The results also support Ruppert's (2013) theoretical model, showing that analogical reasoning develops hierarchically from structuring to verifying. Furthermore, creative thinking significantly influenced the completeness and quality of students' analogical reasoning. Low-ability students relied on intuitive visualization, medium-ability students demonstrated procedural reasoning, and high-ability students exhibited systematic and reflective reasoning. This pattern suggests that creative thinking

enhances students' ability to construct, apply, and evaluate analogical relationships, thereby deepening their mathematical understanding.

1. Relationship Between Creative Thinking and Analogical Reasoning

The results of this study support Tzurriel (2024) and Brookman-Byrne et al. (2019), who emphasize that analogical reasoning contributes to students' mathematical flexibility and creative capacity. The data from this study demonstrates a strong correlation between creative thinking indicators (fluency, flexibility, and originality) and the sophistication of analogical reasoning processes. Students capable of constructing analogical correspondences demonstrate cognitive adaptability by shifting between verbal, visual, and symbolic representations when solving unfamiliar problems. In this research, students with higher creativity (MA) utilized multiple analogies and representations, both geometric and algebraic, to verify their reasoning, showing that creativity fosters reflective thinking and metacognitive control. Conversely, students with lower ability (NF) tended to depend on surface-level similarities, consistent with English's (2004) observation that novice reasoners often focus on superficial analogy mapping rather than structural relationships. Thus, this finding emphasizes that creative and analogical reasoning operate interdependently, with creativity supporting flexibility and analogical reasoning organizing those ideas into coherent conceptual structures.

However, while this study found that highly creative students exhibited the most complete analogical reasoning, previous research offers nuanced perspectives. Kipnis et al. (2019) reported that although gifted students initially performed better on analogy tasks, their improvement rate was comparable to that of average-ability peers under supportive learning conditions. Similarly, Threlfall (2020) noted that students with moderate ability sometimes displayed greater flexibility than high-ability students by exploring multiple solution paths and learning through self-correction. This aligns with Putra et al. (2024), who found that Indonesian students improved their mathematical reasoning and analogical thinking when engaged in contextual, scaffolded learning through the Search, Solve, Create, and Share (SSCS) model. Recent studies further support this, showing that scaffolding and analogical reasoning together enhance mathematical creativity (Nuryadi & Hartono, 2022; Wijayanti, 2023; Zhu et al., 2024). These findings suggest that the completeness of analogical reasoning does not always correlate linearly with cognitive ability but depends on the learning environment and task

openness (Gradini et al., 2025; Palwa et al., 2024). Therefore, open-ended learning contexts such as those applied in this study serve as effective scaffolds that promote both creative and reflective reasoning. This study provides empirical evidence that students' labeling of creative flexibility significantly influences the completeness and structural quality of analogical reasoning in solving open-ended mathematical reflection problems. Specifically, students with high creativity (MA) successfully performed all stages of Ruppert's model and were supported by strong logical coherence, conceptual flexibility, and reflective awareness. Conversely, students with low creative thinking (NF) often relied on intuitive visualization, resulting in superficial mapping based on surface-level features. These findings suggest that creative thinking and analogical reasoning are interdependent, with creative flexibility serving as the essential mechanism for retrieving and utilizing diverse analogies. Analogical reasoning then organizes these creative ideas into coherent conceptual structures.

2. Pedagogical Implications

The findings offer several implications for mathematics instruction. First, integrating analogical reasoning into classroom learning can bridge abstract mathematical concepts with students' real-life experiences. Teachers are encouraged to design activities that require identifying, mapping, and applying structural similarities between mathematical situations. Such activities cultivate conceptual understanding and knowledge transfer. Second, open-ended tasks are essential for fostering creativity. Reflection-based, open-ended problems enable students to explore multiple strategies, demonstrate flexibility, and refine their conceptual understanding through diverse representations. Third, greater emphasis should be placed on the verification stage of problem-solving. Encouraging students to review, evaluate, and justify their reasoning nurtures metacognitive awareness and independent thinking. Ultimately, differentiated instruction is essential to accommodate students' diverse reasoning abilities. Structured prompts and guided analogies can support students with low ability, while exploratory modeling can challenge high-ability students. Thus, this finding emphasizes that targeted scaffolding and open-ended inquiry can enhance both analogical reasoning and creative development in mathematics learning.

3. Theoretical Contributions

Theoretically, this study extends Ruppert's (2013) model of analogical reasoning by showing that progression through its four stages, structuring, mapping, applying, and verifying, is mediated by students' creative thinking levels. The model serves not only as a cognitive framework but also as a diagnostic lens for identifying reasoning strengths and weaknesses. Moreover, these findings enrich Gentner's (1983) structure-mapping theory by demonstrating how structural correspondences emerge in mathematical reasoning tasks that require creativity and reflection. The integration of creative thinking indicators fluency, flexibility, originality, and elaboration with analogical reasoning stages supports a dual-process model of mathematical cognition that connects divergent idea generation with convergent structural reasoning. Thus, this finding emphasizes that creative and analogical reasoning should be viewed not as separate processes but as mutually reinforcing components of higher-order mathematical thought. The findings of this study provide empirical support for Tzuriel (2024) and Brookman-Byrne et al. (2019), who assert that analogical reasoning fosters mathematical flexibility and creative capacity. Crucially, the data extends this theoretical view by demonstrating that high creative flexibility (fluency and originality) actively enables students to achieve sophisticated analogical mapping and verification processes.

CONCLUSION

This study demonstrates that students' levels of creative thinking significantly influence the completeness and quality of analogical reasoning in solving open-ended mathematical reflection problems. Students with high creativity were able to perform all stages of Ruppert's model, including structuring, mapping, applying, and verifying with strong logical coherence, conceptual flexibility, and reflective awareness. Those with medium ability showed partial understanding and procedural reasoning, while students with low creative thinking relied mainly on intuitive visualization without formal abstraction. These findings indicate that innovative thinking and analogical reasoning are interdependent cognitive processes where creativity enables flexible mapping and analogical reasoning organizes creative ideas into coherent structures.

In educational practice, integrating analogical reasoning tasks into open-ended mathematics learning can enhance students' reflective and adaptive thinking while

deepening conceptual understanding and fostering creativity. The limited sample size and focus on geometric reflection restrict the generalization of findings; therefore, future research should include more participants, various mathematical topics, exploring the relationships between creative thinking and analogical reasoning within other mathematical domains, particularly algebra and function topics, designing and testing classroom based interventions focused on systematically developing students analogical reasoning skills, and observational methods to capture reasoning processes dynamically. Overall, this study contributes to mathematics education research by providing empirical insights into how creative and analogical reasoning can be effectively integrated within open-ended learning contexts.

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