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A Nonlinear mathematical model of student motivation dynamics with an empirical calibration framework and simulation analysis

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Abstract

Student learning motivation is a dynamic construct that changes over time in response to support, academic pressure, and fatigue. However, many existing studies examine motivation only at a single point in time or rely on descriptive approaches, limiting their ability to quantify how motivation evolves. This study develops a nonlinear mathematical model of student motivation dynamics and links the model to an empirical calibration framework. The proposed model represents motivation as a bounded process influenced by support saturation, motivational decline, and nonlinear burnout effects. To reduce the analysis's hypothetical nature, the study outlines how repeated measurements of motivation, perceived support, and inhibiting factors can be used to estimate model parameters from longitudinal data. The framework is accompanied by scenario-based simulations under stable support, temporary academic stress, early intervention, delayed intervention, and high-burnout conditions. These scenarios are designed to generate substantive insight into recovery, intervention timing, and motivational vulnerability. The study offers a mathematically interpretable and educationally relevant framework for understanding student motivation over time and for guiding future data-informed intervention design.

Keywords: nonlinear differential equation; empirical calibration; simulation; student motivation; burnout

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INTRODUCTION

Students' learning motivation plays a crucial role in academic engagement, persistence, and achievement in higher education. Previous studies have shown that support from lecturers, peers, and family can strengthen students' willingness to learn, whereas academic pressure, fatigue, and stress can weaken it over time. In this sense, motivation is not a static attribute but a dynamic educational process that continuously changes in response to internal and external conditions (Bureau et al., 2022; Dörnyei & Ushioda, 2021; Nurlaili et al., 2024).

Despite this dynamic character, many motivation studies still rely on cross-sectional surveys or descriptive analyses. These approaches are useful for identifying factors associated with motivation, but they generally provide only a snapshot of students' conditions and do not explain how motivation evolves. As a result, a gap remains between educational reality and the quantitative tools used to represent it. Earlier mathematical modeling studies have begun to address this issue, yet many models remain linear or focus on general learning behavior rather than the nonlinear dynamics of motivation itself (Mutiawati et al., 2022; Deboeck & Bergeman, 2013; Saqr et al., 2023).

The previous study adopted a simple first-order linear model and used assumption-driven simulations. Although that formulation was analytically convenient, it was limited in two important ways. First, the model was too simple to reflect real-world nonlinear behavior, because it treated support as constant and decline as purely proportional. Second, the simulation largely replicated the expected theoretical pattern that larger parameter values lead to faster convergence, offering limited educational insight. These limitations suggest the need for a stronger formulation that captures bounded growth, diminishing returns of support, and burnout-related decline, while also moving closer to an empirical framework.

Accordingly, this study develops a nonlinear mathematical model of student motivation dynamics and complements it with an empirical calibration framework. The model treats motivation as a bounded process influenced by support and inhibiting factors, and by nonlinear burnout, consistent with the view that motivational processes are adaptive and context-dependent (Ryan & Deci, 2020). In addition, the study proposes how repeated measurements of motivation, support, and inhibiting factors can be used to estimate model parameters from longitudinal data. This approach helps reduce the analysis's hypothetical nature and gives simulations a more substantive role.

The objectives of this study are threefold. First, it reformulates the linear equation as a nonlinear model that reflects real motivational processes. Second, it presents an empirical calibration framework that links the model to observable student data. Third, it redesigns the simulation analysis around meaningful educational scenarios, including stress shocks, intervention timing, and burnout. Therefore, the study aims to provide a more realistic and more informative account of student motivation dynamics.

METHODS

This study employs a quantitative mathematical modeling approach with an explicit empirical calibration framework. The purpose of this design is twofold: to formulate a nonlinear dynamic model of student motivation and to specify how the model can be linked to repeated observations of students' motivational conditions. In contrast to a purely theoretical approach, the revised framework positions model parameters as estimable quantities derived from longitudinal measurements rather than arbitrary constants.

Let $M_i(t)$ denote the motivation level of student *i* at time *t*, $S_i(t)$ the support index, and $D_i(t)$ the inhibition index. In an empirical implementation, motivation can be measured using a validated questionnaire and normalized to a 0–100 scale. Support may be constructed from indicators such as lecturer guidance, peer encouragement, family support, and learning environment quality, whereas inhibiting factors may be constructed from academic workload, fatigue, stress, and distraction. Repeated measurements across several weeks or academic periods allow the observed trajectories of these variables to be used for model calibration.

$$\frac{dM_i}{dt} = \alpha S_i \left(1 - \frac{M_i}{K}\right) - \beta D_i M_i - \gamma M_i^2$$

In this formulation, *K* denotes the maximum attainable motivation level, α represents the contribution of support to motivational growth, *β* measures the effect of inhibiting factors on motivational decline, and γ captures nonlinear burnout. The factor $\left(1 - \frac{M_i}{K}\right)$ introduces a saturation effect, implying that the same amount of support produces a smaller gain when motivation is already high. The quadratic term γM_i^2 reflects the possibility that motivational depletion accelerates under sustained effort.

In a future empirical application, the parameters α, β, γ, and *K* can be estimated from longitudinal data using nonlinear least squares or related estimation procedures. The estimated nonlinear model can then be compared with a simpler linear model by using goodness-of-fit measures such as RMSE, MAE, or information criteria. This calibration step is essential because it transforms the model from a purely hypothetical system into a data-informed representation of student motivation.

After calibration, the model can be used for scenario-based simulation. In this study, the simulations are organized around educationally meaningful cases. The scenarios include stable support, temporary academic stress shocks, early support intervention, delayed support intervention, and high-burnout conditions. The resulting trajectories are interpreted in terms of minimum motivation, depth of decline, recovery time, and long-term stabilization.

RESULTS AND DISCUSSION

Nonlinear Mathematical Model Formulation

The model formulates the nonlinear differential equation by allowing support to exhibit diminishing returns and by allowing decline to intensify under burnout. Mathematically, the equilibrium condition is obtained by setting. $\frac{dM_i}{dt} = 0$, which yields a nonlinear balance between growth and decline. Educationally, this means that support is beneficial but not infinitely effective, and motivational loss is not always proportional.

The nonlinear structure also provides a clearer interpretation of motivational behavior. When support dominates, and motivation remains far from its upper bound, the trajectory tends to rise. As inhibiting factors and burnout intensify, the trajectory may flatten, decline, or recover more slowly. Thus, the model can represent not only movement toward equilibrium but also the sensitivity of that movement to timing and pressure.

Empirical Calibration Framework

The paper explicitly links the model to observable data. In practice, motivation, support, and inhibiting factors should be collected repeatedly from the same group of students across time. This allows each motivational trajectory to be compared with the model-generated trajectory, enabling the parameters α, β, γ, and K to be estimated rather than assumed. This empirical calibration framework does not require abandoning the model's mathematical structure. Instead, it strengthens the study by making the parameters interpretable in educational terms. A larger estimated α would indicate stronger responsiveness to support, whereas a larger β or γ would indicate greater vulnerability to pressure and burnout. In addition, comparing the nonlinear model with a linear baseline would make it possible to examine whether the nonlinear terms truly improve explanatory power.

The calibration procedure makes the study more empirically oriented and provides a clear direction for future implementation. In this way, the paper moves beyond a purely assumption-based model and offers a framework that can be tested with real longitudinal data.

Scenario-Based Simulation Analysis

The scenarios based on simulation focus on educational situations with practical meaning. These scenarios include stable support, a temporary academic stress shock, early support intervention, delayed intervention, and a high-burnout condition. Such scenarios do not simply show faster or slower convergence; they reveal how timing and pressure influence motivational recovery and vulnerability.

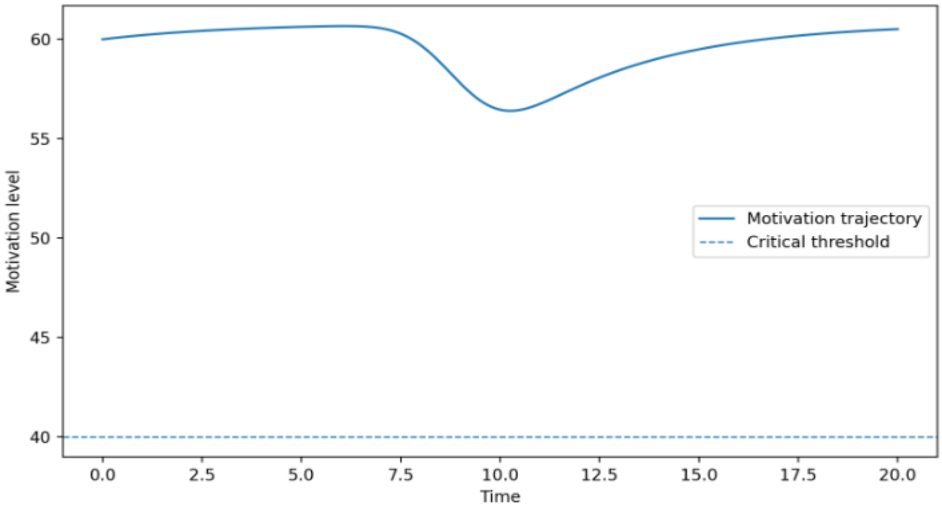

Figure 1. Temporary Academic Stress Shock

Figure 1 shows the temporary academic stress scenario. The trajectory declines during the stress period and then recovers gradually after the shock subsides. This result suggests that temporary academic overload may have effects that persist beyond the stressful period itself.

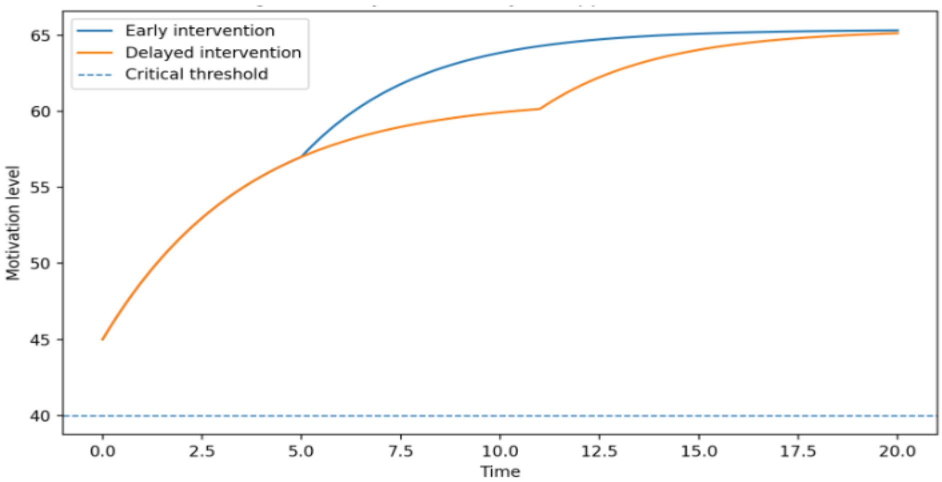

Figure 2. Early versus Delayed Support Intervention

Figure 2 compares early and delayed support intervention. The early intervention trajectory recovers faster and remains higher over time than the delayed intervention trajectory. This indicates that the timing of support matters, not only its magnitude. In practice, proactive support is likely to be more effective than waiting until motivation has already fallen considerably.

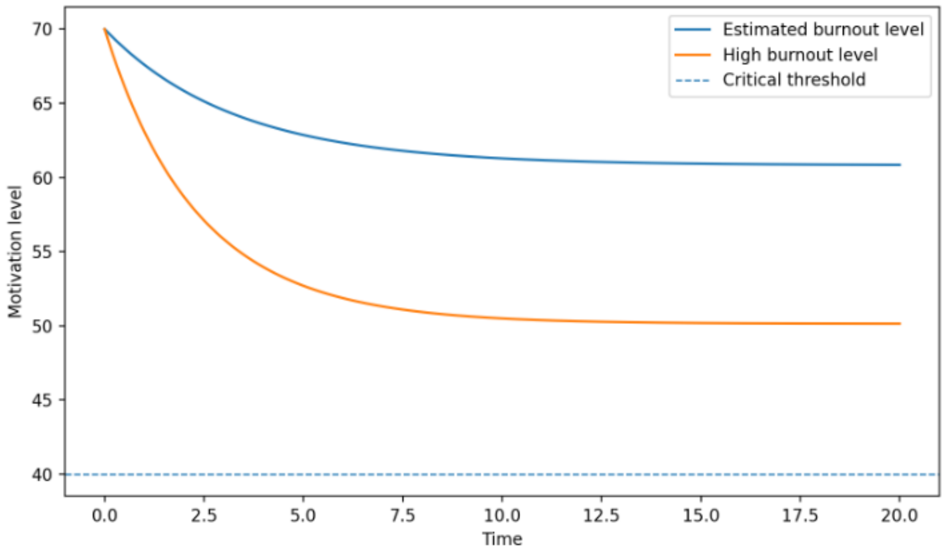

Figure 3. Motivation under High-Burnout Condition

Figure 3 illustrates the high-burnout condition. Even under comparable support and inhibition conditions, greater burnout is associated with a lower motivational trajectory. This implies that support alone may not be sufficient if fatigue and sustained pressure accumulate too quickly.

Taken together, the simulations provide the following insights. First, temporary stress can produce a recovery lag even after conditions begin to improve. Second, timely support is more effective than delayed support. Third, high initial motivation is not always protective when burnout becomes strong. These findings show that simulation analysis contributes to interpretive value. This study provides a nonlinear and potentially data-calibrated framework for representing student motivation. The model remains analytically interpretable while being flexible enough to incorporate real educational scenarios and support future longitudinal estimation. Therefore, the study provides a stronger bridge between mathematical formulation and educational application.

CONCLUSION

This study proposes a nonlinear mathematical model of student motivation dynamics that is more realistic than the linear formulation. By incorporating bounded growth, support saturation, and burnout-related decline, the model better reflects the nonlinear behavior observed in real educational settings. The paper also presents an empirical calibration framework based on repeated measurements of motivation, support, and inhibiting factors. It clearly specifies how the parameters can be estimated from longitudinal data, thereby moving the study closer to a data-informed approach. Finally, the simulation analysis generates substantive educational insight. The scenarios demonstrate the effects of temporary stress, intervention timing, and burnout on motivational trajectories. Future studies are encouraged to implement the calibration procedure using real student data and to extend the model to include peer interaction, adaptive feedback, or individual heterogeneity.

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DECLARATIONS

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Conflict of Interest	:	The authors declare that there are no conflicts of interest related to the research, writing, or publication of this article.
Additional Information	:	Supplementary materials, including simulation data, program files, and supporting documents, are available from the corresponding author upon request.

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